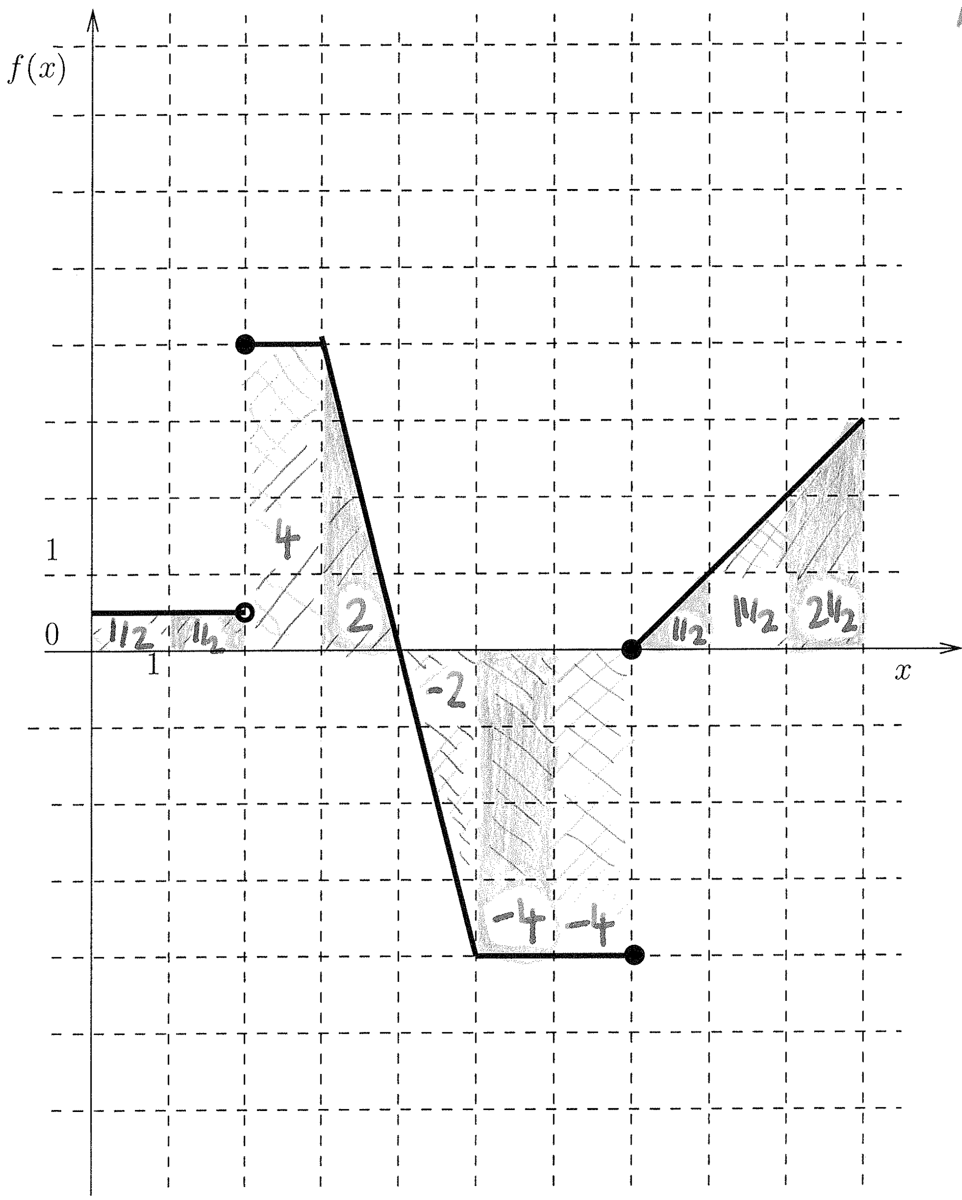
STUDENT NAME: THIS SHEET SHOWS QUESTION

Calculus 1550, section 6. Tuesday, April 20, 2004. Eighteenth quiz.

Below the graph of a function $f(x)$ is sketched.

1. [6 points] On the same grid, Sketch a graph of the function

$$g(x) = \int_0^x f(t)dt$$



Areas under each interval of graph are shown on graph, so have following values:

x	$g(x)$
0	~~½~~ 0
1	½ + ½ = 1
2	1 + 4 = 5
3	
4	5 + 2 = 7
5	7 − 2 = 5
6	5 − 4 = 1
7	1 − 4 = −3
8	−3 + 1½ = −2½
9	−2½ + 1½ = −1
10	−1 + 2½ = 1½.

these values can be plotted & connected to give a graph of g(x). (See other sheet)

2. [1.5 point] What is the maximum value of $g(x)$ on $[0, 10]$, and where is it achieved?

3. [1.5 point] What is the minimum value of $g(x)$ on $[0, 10]$, and where is it achieved?

4. [1 point] Suppose

$$\int_a^x f(t)dt = g(x) - 5$$

What is a possible value of a?

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Below the graph of a function $f(x)$ is sketched.

1. [6 points] On the same grid, Sketch a graph of the function

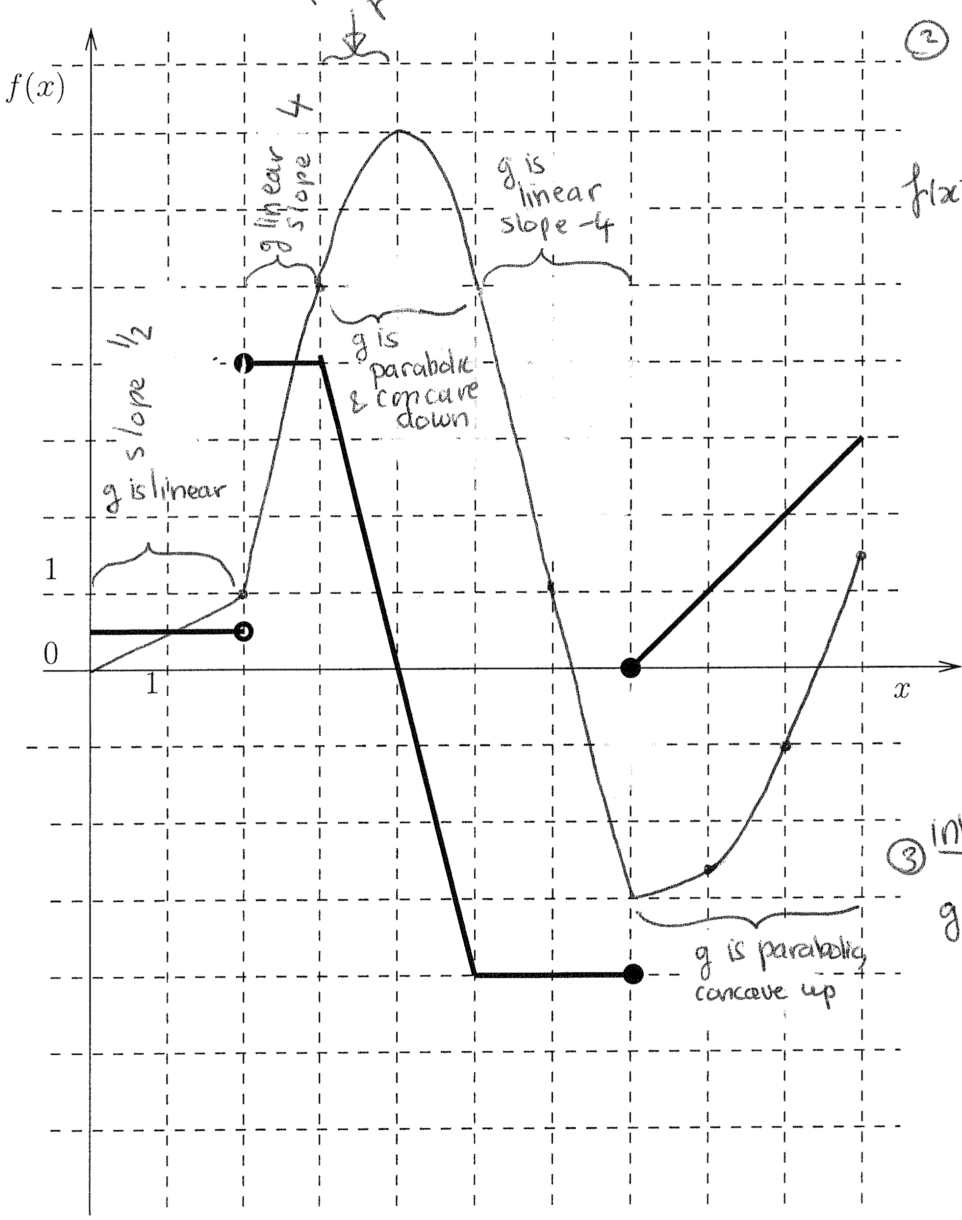
$$g(x) = \int_0^x f(t)dt$$

g still increasing here, because f is +ve

further comments:

① note, g is continuous, even though f is not continuous.

② f is given by



$$f(x) = \begin{cases} 1/2 & x \in [0,2] \\ 4 & x \in [2,3] \\ -4(x-4) & x \in [3,5] \\ -4 & x \in [5,7] \\ x-7 & x \in [7,10] \end{cases}$$

slope $= -4$, intercept $(4,0)$

(slope $= 1$ & when $x = 7$, $f(x) = 0$)

③ integral:

$$g(x) = \begin{cases} 1/2\, x & x \in [0,2] \\ 4(x - 1\tfrac{3}{4}) & x \in [2,3] \\ -2(x-4)^2 + 7 & x \in [3,5] \\ -4(x - 6\tfrac{1}{4}) & x \in [5,7] \\ \tfrac{1}{2}(x-7)^2 - 3 & x \in [7,10] \end{cases}$$

(because max value is 7)

2. [1.5 point] What is the maximum value of $g(x)$ on $[0, 10]$, and where is it achieved?

check that $g'(x) = f(x)$

3. [1.5 point] What is the minimum value of $g(x)$ on $[0, 10]$, and where is it achieved?

4. [1 point] Suppose

$$\int_a^x f(t)dt = g(x) - 5$$

④ note, where $f(x) = 0$, $g(x)$ has a local max or min.

What is a possible value of a?

⑤ note, if $f'(x) > 0$, eg, on $[7,10]$, $g''(x) = f'(x) > 0 \Rightarrow g$ is concave up

on $[3,5]$, $f'(x) < 0$, so g is concave down