STUDENT NAME:

Calculus 1550, section 6. Wednesday, April 28, 2004. Ninteenth quiz.
(worth 20 out of 10 point.)

1. Let $f(x)$ be the function

$$f(x) = \frac{x^2}{e^{(x-2)}}.$$

i. [1 point] What is the domain of f? $\mathbb{R}$, since x^2 has domain $\mathbb{R}$, $e^{(x-2)}$ has domain $\mathbb{R}$ & $e^{x-2} > 0$ for all $x \in (-\infty, \infty)$

ii. [1 point] What are the x and y intercepts of $f(x)$, if any? $(0,0)$

if $x=0$, $y=0$ & $y=0 \Rightarrow x=0$

iii. [1 point] Is f odd, even, or periodic, or have no symmetries? no symmetry

iv. [1 point] Does f have vertical asymptotes, and if so, what are they? none

since denominator is never 0

v. [1 point] Using L'hopital's rule, or otherwise, compute $\lim_{x\to\infty} f(x)$.

$$\lim_{x\to\infty} \frac{x^2}{e^{(x-2)}} = \lim_{x\to\infty} \frac{2x}{e^{x-2}} = \lim_{x\to\infty} \frac{2}{e^{x-2}} = 0$$

vi. [1 point] Using L'hopital's rule, or otherwise, compute $\lim_{x \to -\infty} f(x)$.

$$\lim_{x \to -\infty} x^2 = \infty \quad \& \quad \lim_{x \to -\infty} \frac{1}{e^{(x-2)}} = \infty,$$

$$\left(\text{since } \lim_{x \to -\infty} e^{x-2} = 0\right)$$

$$\text{so } \lim_{x \to -\infty} f(x) = \infty$$

vii. [1 point] Does f have horizontal asymptotes, and if so, what are they? $y = 0$

$$\left(\text{since } \lim_{x \to \infty} f(x) = 0\right)$$

viii. [1 point] What is the derivative of $f(x)$?

$$f(x) = \frac{x^2}{e^{x-2}} \Rightarrow f'(x) = \frac{e^{x-2} \times 2x - x^2 e^{x-2}}{(e^{x-2})^2}$$

$$= \frac{\cancel{e^{x-2}}\, x(2-x)}{(e^{x-2})^{\cancel{2}}} = \frac{x(2-x)}{e^{x-2}}$$

ix. [1 point] What are the critical numbers of $f(x)$, if any? 0 & 2

$$\left(\text{since at these values, } f'(x) = 0\right)$$

	$(-\infty, 0)$	0	$(0, 2)$	2	$(2, \infty)$
x	$-$	0	$+$	$+$	$+$
$2-x$	$+$	$+$	$+$	0	$-$
e^{x-2}	$+$	$+$	$+$	$+$	$+$
$f'(x)$	$-$	0	$+$	0	$-$
		local min		local max	

x. [1 point] What are the values of $f(x)$ at local maximum and local minimum?

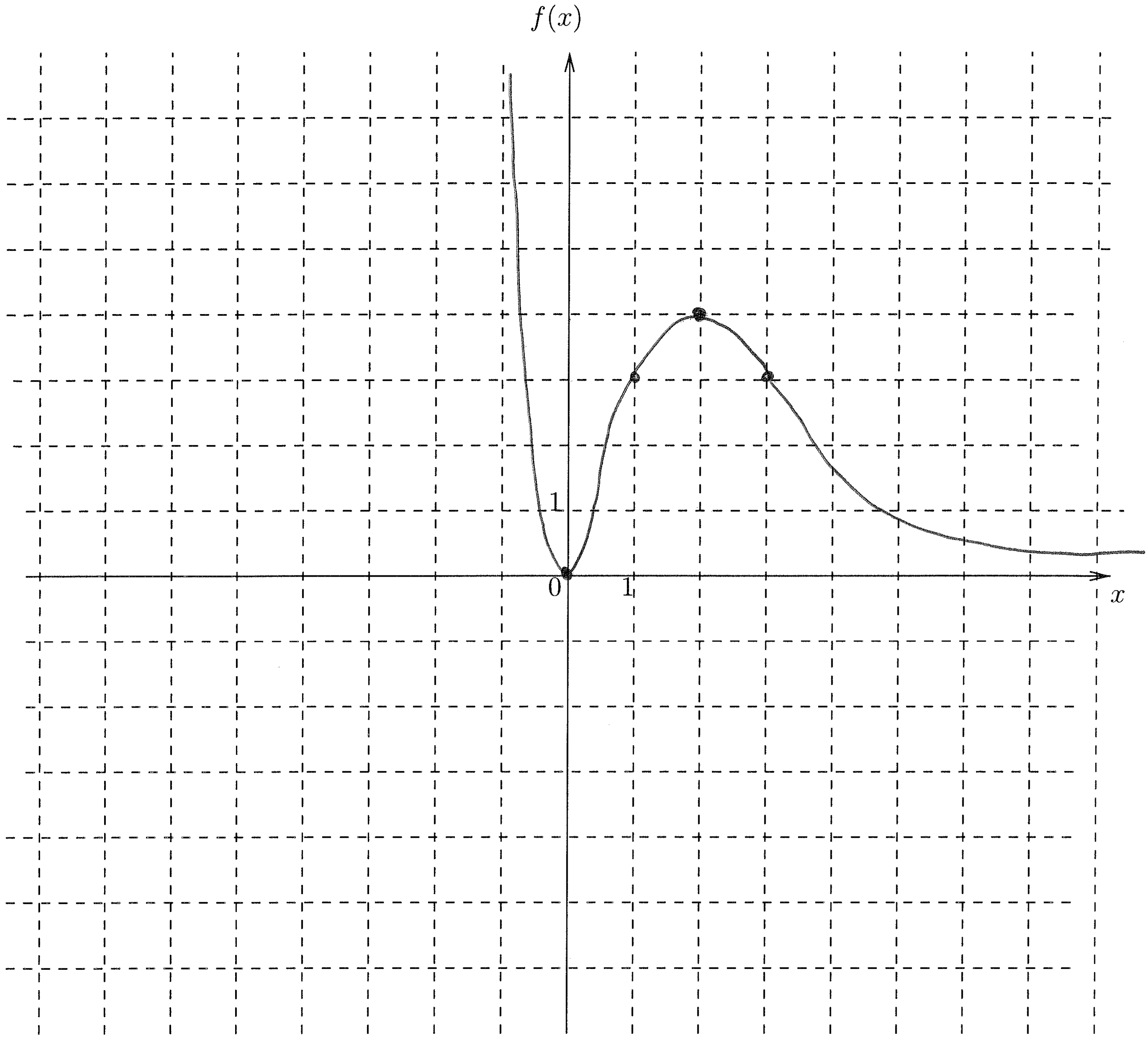
$$f(0) = \frac{0}{e^{-2}} = 0, \quad f(2) = 2^2/e^{2-2} = 4$$

$(0, 0)$ is a local min

$(2, 4)$ is a local max

STUDENT NAME:

xi. [10 points] Sketch a graph of $f(x)$, showing all the above features.
[Note, for rough approximations for plotting points, you may use $e \approx 3$, so, e.g., $9/e \approx 3$.]



$$f(3) = \frac{3^2}{e^{3-2}} = \frac{9}{e} \approx 3$$

$$f(-1) = 1/e^{-3} \approx 1/3^{-3} = 3^3 = 27$$

$$f(1) = 1/e^{-1} = e \approx 3$$

$$f(4) = 4^2/e^2 \approx 16/9 \approx 1.8$$