

# STUDENT NAME:

Calculus 1550, section 6. Wednesday, February 18, 2004. Seventh quiz.

Find the following limits at infinity. [2 points each].

1.  $\lim_{x \rightarrow \infty} \left( \frac{x^3 + x - 1}{x - 1} + 2 \right) = \infty$  because degree of denominator is greater than degree of numerator & all coefficients are +ve.

more formally:  $\lim_{x \rightarrow \infty} \left( \frac{x^3 + x - 1}{x - 1} + 2 \right) = \lim_{x \rightarrow \infty} \left( \frac{x + 1 + 1/x}{1 - 1/x} \right) + 2$

but then  $\lim_{x \rightarrow \infty} x + 1 + 1/x \rightarrow \infty$ ,  $\frac{\infty}{1} = \infty$ , so  $\lim_{x \rightarrow \infty} \left( \frac{x + 1 + 1/x}{1 - 1/x} \right) = \infty$

&  $\lim_{x \rightarrow \infty} 1 - 1/x = 1$

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2.  $\lim_{x \rightarrow -\infty} \left( \frac{x - 1}{x^3 + x - 1} + 2 \right) = 2$  working:  $\lim_{x \rightarrow -\infty} \left( \frac{x - 1}{x^3 + x - 1} + 2 \right) = \lim_{x \rightarrow -\infty} \left( \frac{x - 1}{x^3 + x - 1} \right) + 2$

$= \lim_{x \rightarrow -\infty} \left( \frac{1/x^2 - 1/x^3}{1 + 1/x^2 - 1/x^3} \right) + 2 = \frac{0}{1} + 2 = 2$

3.  $\lim_{x \rightarrow \infty} \frac{3x^3 + x - 1}{4x^3 - 1} = \frac{3}{4}$  working:  $= \lim_{x \rightarrow \infty} \frac{3x^3 + x - 1}{4x^3 - 1} = \lim_{x \rightarrow \infty} \left( \frac{3 + 1/x^2 - 1/x^3}{4 - 1/x^3} \right)$

$= \frac{\lim_{x \rightarrow \infty} (3 + 1/x^2 - 1/x^3)}{\lim_{x \rightarrow \infty} (4 - 1/x^3)} = \frac{3}{4}$

4.  $\lim_{x \rightarrow \infty} \frac{|x^3 - 1|}{x^3 - 5} = 1$  working:  $\lim_{x \rightarrow \infty} \frac{|x^3 - 1|}{x^3 - 5} = \lim_{x \rightarrow \infty} \frac{\frac{1}{|x^3|} |x^3 - 1|}{\frac{1}{x^3} (x^3 - 5)} = \lim_{x \rightarrow \infty} \frac{|1 - 1/x^3|}{1 - 5/x^3} = \frac{1}{1} = 1$

OR when  $x$  +ve,  $\frac{|x^3 - 1|}{x^3 - 5} = \frac{x^3 - 1}{x^3 - 5}$  - treat as usual.

because  $|x^3| = x^3$  for  $x > 0$

5.  $\lim_{x \rightarrow -\infty} \frac{|x^3 - 1|}{5 - x^3} = 1$  working:  $\lim_{x \rightarrow -\infty} \frac{|x^3 - 1|}{5 - x^3} = - \lim_{x \rightarrow \infty} \frac{\frac{1}{|x^3|} |x^3 - 1|}{\frac{1}{x^3} (5 - x^3)} = - \lim_{x \rightarrow \infty} \frac{|1 - 1/x^3|}{5/x^3 - 1}$

$= - \frac{1}{-1} = \frac{1}{1} = 1$

( $|x^3| = -x^3$  for  $x$  -ve)

OR when  $x$  -ve,  $x^3$  is -ve,

so  $\frac{|x^3 - 1|}{5 - x^3} = \frac{-x^3 + 1}{5 - x^3} = \frac{-x^3 + 1}{-x^3 + 5} \approx \frac{-x^3}{-x^3} = 1$

for large  $x$