Q1. A function $f(x)$ is defined by

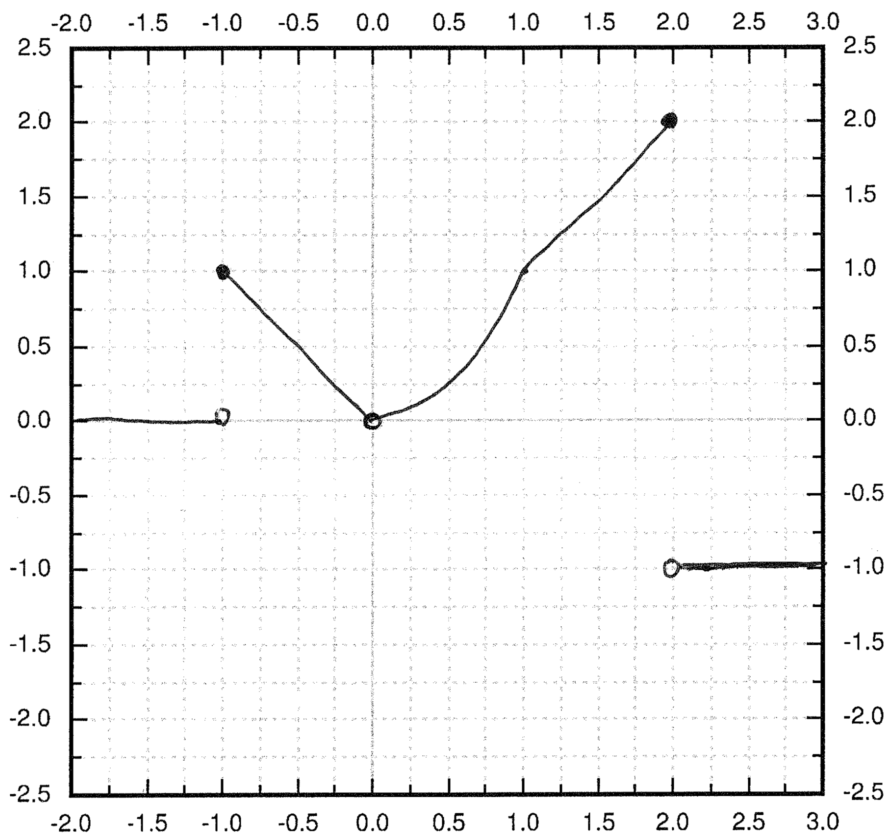
$$f(x) = \begin{cases} 0 & \text{if } x < -1 \\ -x & \text{if } x \in [-1, 0) \\ x^2 & \text{if } x \in (0, 1) \\ x & \text{if } x \in [1, 2] \\ -1 & \text{if } x > 2 \end{cases}$$

i. [5points]

a) Where is $f(x)$ discontinuous? $x = -1$ & 2 & 0 (jump discontinuity at -1 & 2) (removable discontinuity at 0)

b) Where is $f(x)$ not defined? $x = 0$

ii. [5 points] Sketch the graph of $f(x)$ on the following grid:



iii. [15] Complete the following table:

a	$f(a)$	$\lim_{x\to a^-} f(a)$	$\lim_{x\to a^+} f(a)$	$\lim_{x\to a} f(a)$	right continuous at a?	left continuous at a?
-1	1	0	1	Does not exist	yes	no
0	not defined	0	0	0	no	no
2	2	2	-1	Does not exist	no	yes

Q2. Let C be a curve, sketched on the next page, and defined by the equation $y = f(x)$ with

$$f(x) = \sin\left(\frac{\pi}{3}x\right) + x$$

Let P, Q_1 and Q_2 be points on C, with x coordinates 1.5, 0 and 2.5 respectively.

i. [3 points] Plot P, Q_1 and Q_2 on the graph.

ii. [3 points] Sketch the secant from P to Q_1, the secant from P to Q_2, and the tangent at P.

iii. [5 points] What is the slope of the secant from P to Q_1?

$1/2$

sorry, should be other order!

iv. [5 points] What is the slope of the secant from P to Q_2?

$\frac{2 \cdot 5}{1 \cdot 5} = \frac{5/2}{3/2} = 5/3 = 1\,2/3$

v. [4 points] From the answers to part ~~ii~~ iii and ~~iii~~ iv, what can you say about the slope of the tangent at P? I.e., write two numbers that it lies between.

let m = slope of tangent at 1.5. then

$1/2 < m < 1\,2/3$

vi. [4 points] Using your sketch of the tangent on the graph, make an estimate of the slope of the tangent at P, making it clear how you compute your estimate.

estimate slope of tangent $= \frac{1}{1}$ from triangle in diagram, on tangent line

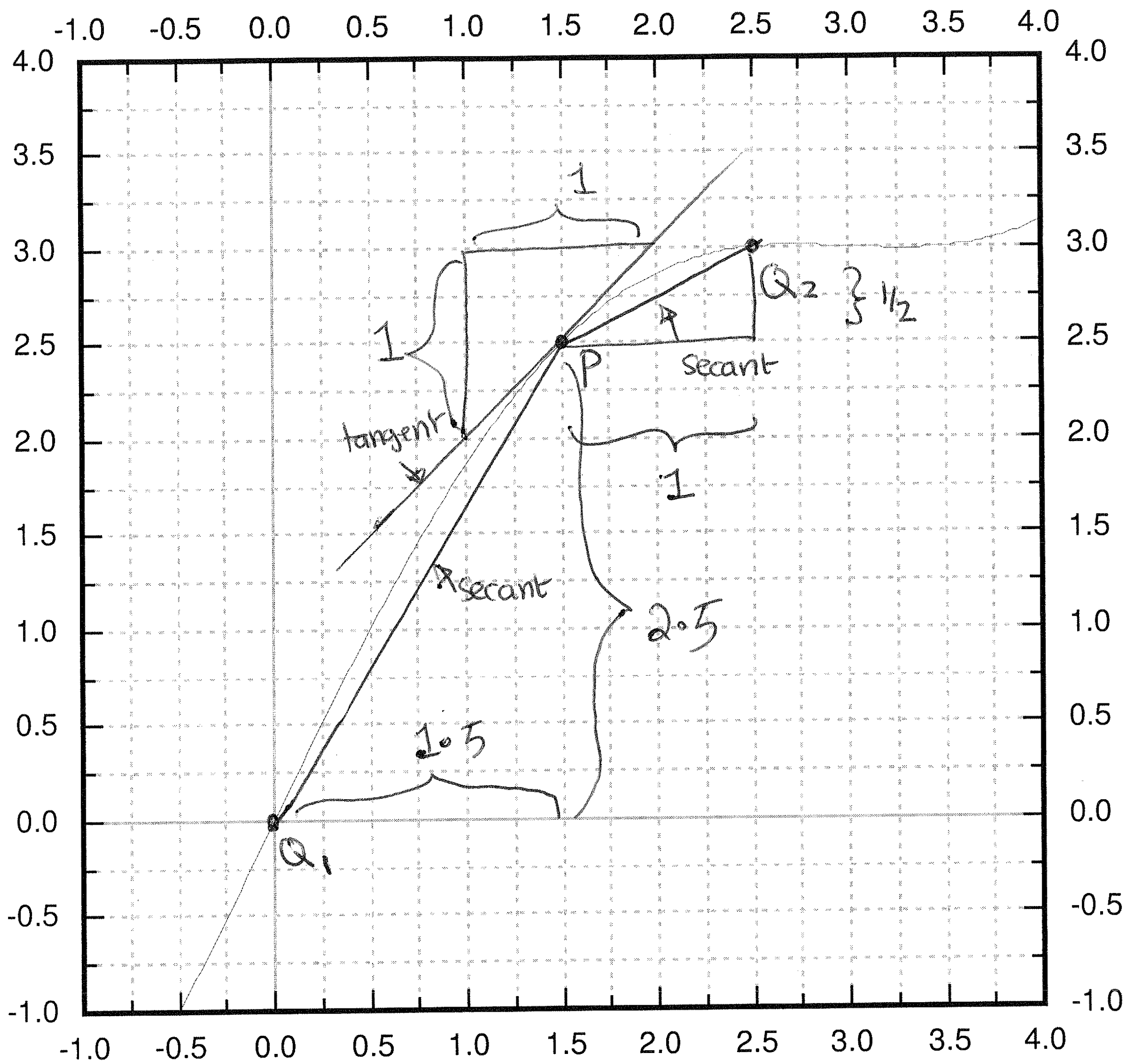
vii [6 points] Using your answer to part vi, write the equation for the tangent at P. $P = (1.5, 2.5)$

$\frac{y - 2.5}{x - 1.5} = 1 \qquad y - 2.5 = x - 1.5$

$y = x + 1$

[later we will see that the slope of the tangent is $\frac{1}{3}\cos(\pi/2) \times \pi + 1 = 1$ at $x = 1.5$]

[note, this does not have to be exactly the right equation, as long as you use your estimate from part ~~iv~~ vi, even if that is not perfect.]



-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
1
1
tangent
P
Q2
1/2
secant
1
secant
2.5
1.5
Q1

Q3. Evaluate the following limits, using limit laws and techniques for computing limits exactly. Show your working, or explain your reasoning, but it is not necessary to write down which laws you are using. If a limit does not exist, explain why not. [5 points each]

i. $\lim_{x\to 2} \frac{x^2-2}{x^3-2}$ $= \frac{4-2}{8-2} = \frac{2}{6}$

ii. $\lim_{x\to 2} \frac{x^3-8}{x^2-10x+16}$ $= \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x-8)} = \frac{4+4+4}{2-8} = \frac{12}{-6} = -2$

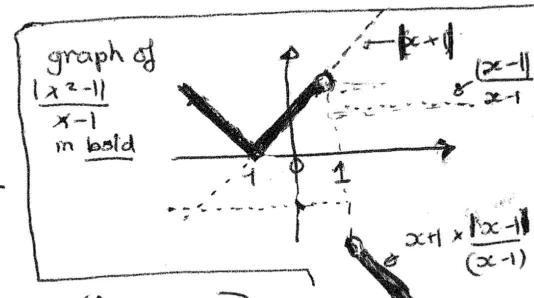
iii. $\lim_{x\to 0} \frac{(x^2-2)^2-x^4}{x^2}$ $= \lim_{x\to 0} \frac{\cancel{x^4} - 4x^2 + 4 - \cancel{x^2}}{x^2} = \lim_{x\to 0}\left(-4 + \frac{4}{x^2}\right) = \infty$

or - does not exist,

(as numerator $x^2 \to 0$)

iv. $\lim_{x\to 0} \frac{\sqrt{x^2+3x}}{x}$ $= \lim_{x\to 0} \sqrt{\frac{x^2+3x}{x^2}} = \sqrt{\lim_{x\to 0} \frac{x^2+3x}{x^2}} = \sqrt{\lim_{x\to 0}\left(1+3/x\right)}$ does not exist,

(as $\lim_{x\to 0} 1/x$ does not exist



v. $\lim_{x\to 1} \frac{|x^2-1|}{x-1}$ Does not exist, because of jump discontinuity - graph looks like ... $|x+1| \times \frac{|x-1|}{(x-1)}$ ...

algebraically: $\lim_{x\to 1+} \frac{|x^2-1|}{x-1} = \lim_{x\to 1+} \frac{x^2-1}{x-1} = \lim_{x\to 1+} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = 2$

$\lim_{x\to 1-} \frac{|x^2-1|}{x-1} = \lim_{x\to 1-} \frac{-\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = -2$

not equal, so $\lim_{x\to 1} \frac{|x^2-1|}{x-1}$ does not exist

Q4.
The graph of the following function is sketched below:

$$f(x) = \frac{x-6}{-6x-4}$$

1. [2 points] If $f(a) = -1$, what is a? $a = -2$

2. [2 points] Sketch the three lines $y = -1$, $y = -1 - \frac{1}{2}$ and $y = -1 + \frac{1}{2}$ on the graph.

3. [4 points] What is the minimum value of x such that $|f(x) + 1| \leq 0.5$? -4

4. [4 points] What is the maximum value of x such that $|f(x) + 1| \leq 0.5$? -1.5

5. [4 points] Write down the set of all ~~positive~~ values of x where $|f(x) + 1| < 0.5$. $(-4, -1.5)$

6. [4 points] Write a number $\delta > 0$ such that $|f(x) + 1| < 0.5$ whenever $|x - a| < \delta$

$\delta = 0.5$