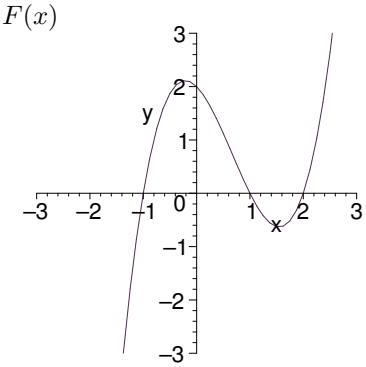
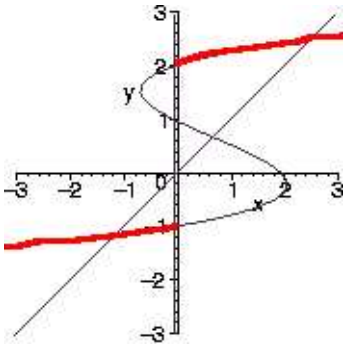
Transformations of a cubic, $F(x) = (x-1)(x-2)(x+1)$

$F(x)$ and inverse | change x | change y

$F(x)$

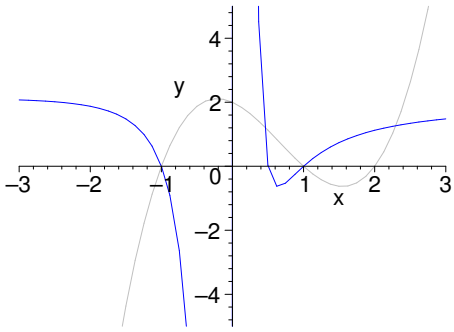


$F^{-1}(x)$

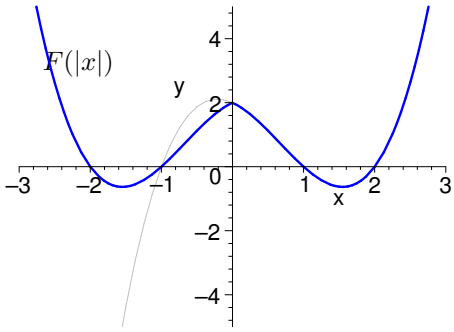


Note that there are many possible ways to take the inverse; they will all have jump discontinuities.
In this picture the inverse is in red, and is obtained by reflecting the graph of $F(x)$ about the line $x = y$, and then just taking one choice of y for each x

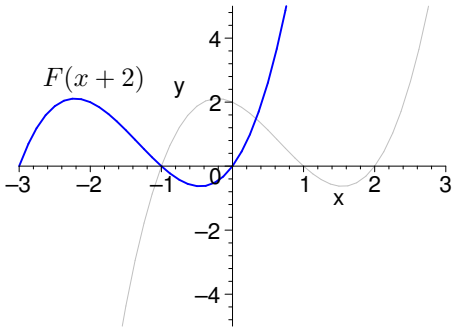
$F(1/x)$



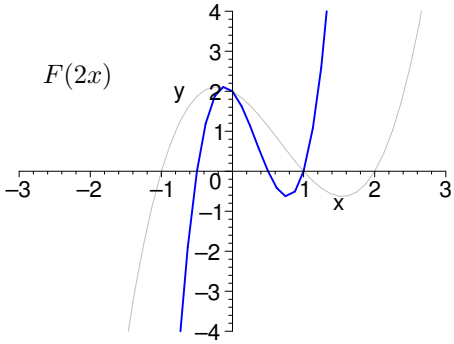
$F(|x|)$



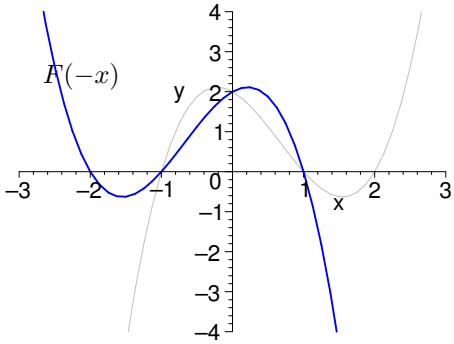
$F(x+2)$



$F(2x)$

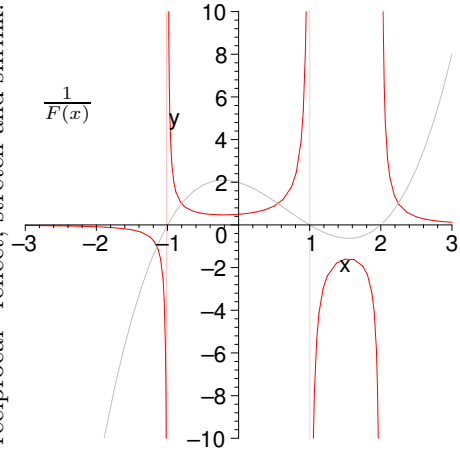


$F(-x)$



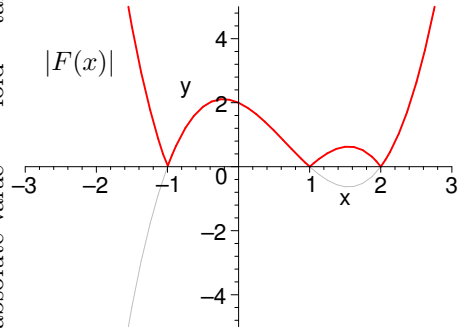
take reciprocal –reflect, stretch and shrink!

$\frac{1}{F(x)}$



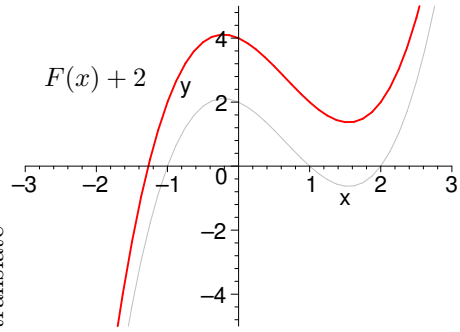
absolute value — "fold"

$|F(x)|$



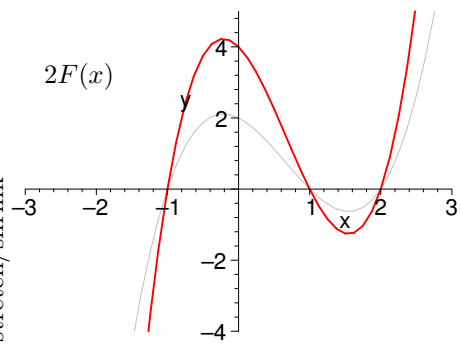
translate

$F(x)+2$



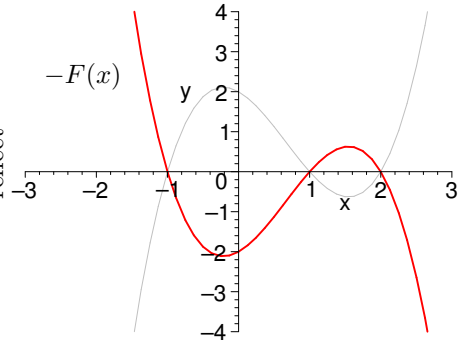
stretch/shrink

$2F(x)$



reflect

$-F(x)$



Exercise:
Write the transformations as rational functions in x
E.g.,
$F(1/x) = \left(\frac{1}{x}-1\right)\left(\frac{1}{x}-2\right)\left(\frac{1}{x}+1\right)$
$= \frac{(1-x)(1-2x)(1+x)}{x^3}$

The graph $F(x)$ appears in grey in each graph, to compare with the transform.