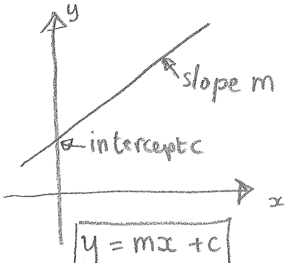
# Graphing:

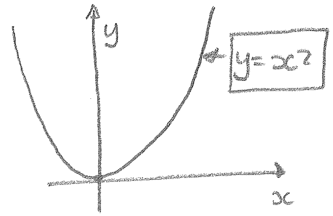
See also pull out sheets in front & back of text book.

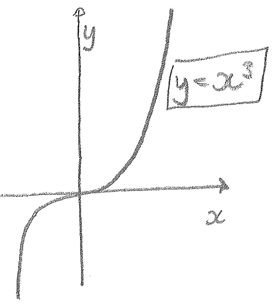
You should know graphs of basic functions:



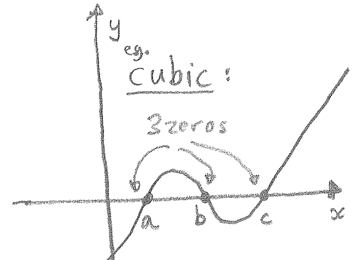

$y = mx + c$

linear functions



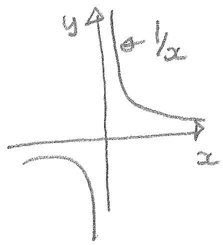

polynomial functions

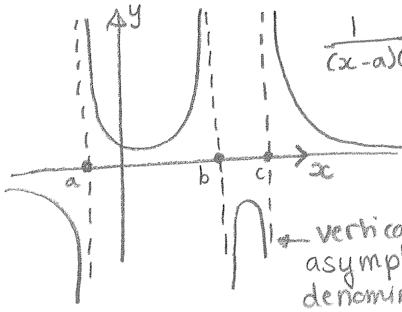



$y = (x-a)(x-b)(x-c)$

generally, a polynomial of degree n has n−1 "bumps", and n zeros (but could have fewer)

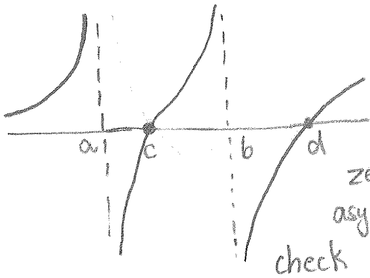
rational functions




$\frac{1}{(x-a)(x-b)(x-c)}$

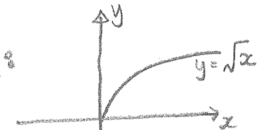
vertical asymptotes when denominator is zero.

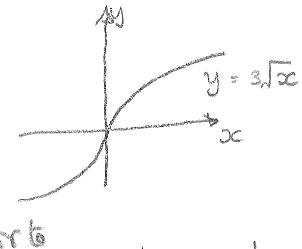



$\frac{(x-c)(x-d)}{(x-a)(x-b)}$

zeros where numerator is 0
asymptotes where denominator is 0
check sign by substitution.

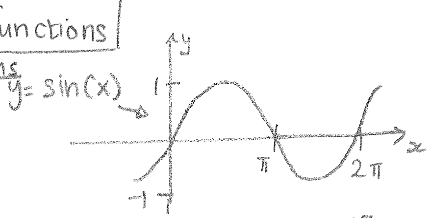
roots:

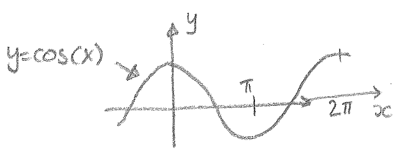


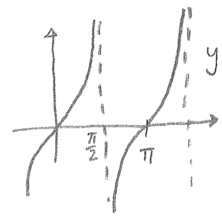




Use your graphing calculator to experiment & get an idea of how other graphs look.

trig functions
in radians

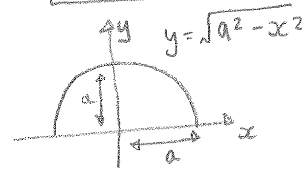


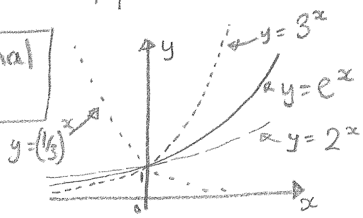




$y = \tan(x) = \frac{\sin(x)}{\cos(x)}$

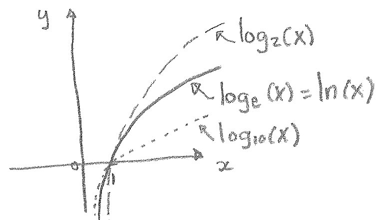
half circle




exponential functions

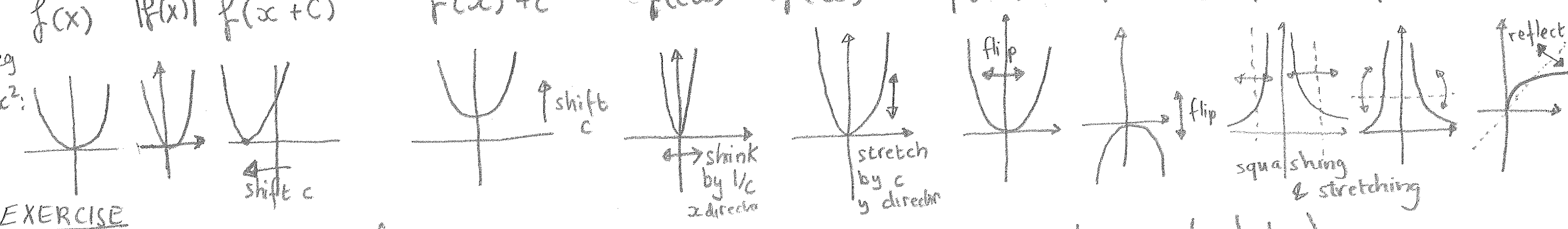



logarithmic functions




Transformations If you know the graph of f(x), you should be able to find the graph of:

$f(x)$ $|f(x)|$ $f(x+c)$ $f(x)+c$ $f(cx)$ $cf(x)$ $f(-x)$ $-f(x)$ $f(\frac{1}{x})$ $\frac{1}{f(x)}$ $f^{-1}(x)$



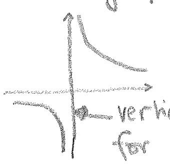

EXERCISE
Try all of these transformations on $\sin(x)$, $e^x$, $x^3$ (use your graphing calculator)

Without using a graphing calculator, you should be able to sketch graphs of:

$\sin(3x)$, $\frac{x^2-3}{x}$, $e^{x+1}-2$, $x\cos(1/x)$, $-x+3$, $\frac{1}{\cos(2x)}$, $\tan(x+\pi)$, $|x^2-2x+1|$

# Asymptotes

**Vertical:** where $\lim_{x\to a} f(x) = \infty$
the line $x = a$ is a vertical asymptote



vertical asymptote, $x=0$ for $y = 1/x$

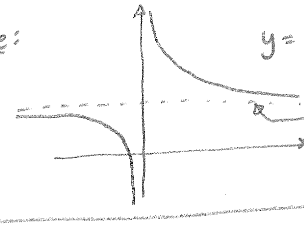
Already covered in section 2.2 pg 99+

## 2.6 Horizontal Asymptote

or if $\lim_{x\to -\infty} f(x) = L$
if $\lim_{x\to\infty} f(x) = L$
with $L \in \mathbb{R}$
$y = L$ is a horizontal Asymptote

WHAT IT IS:

example: $y = \frac{1}{x} + 1$



$y=1$, horizontal asymptote

**What limit at ∞ means:**

$\lim_{x\to\infty} f(x) = L$ means as $x$ gets arbitrarily big, $f(x)$ can be made arbitrarily close to $L$

$\lim_{x\to-\infty} f(x) = L$ means by making $x$ sufficiently large negative, $f(x)$ can be made arbitrarily close to $L$

how to find horizontal asymptotes & limits at $\pm\infty$

- use a table - just plug a large number in
- limit laws
  same as in 2.3, plus for a +ve integer $n$,

$$\lim_{x\to\infty} \frac{1}{x^n} = 0 \qquad \lim_{x\to\infty} x^n = \infty$$

$$\lim_{x\to-\infty} \frac{1}{x^n} = 0 \qquad \lim_{x\to-\infty} x^n = \begin{cases} \infty & \text{if } n \text{ even} \\ -\infty & \text{if } n \text{ odd} \end{cases}$$

polynomials - never have horizontal asymptotes (unless they are constant eg $f(x) = 2$)

rational functions: have horizontal asymptotes when the degree of the numerator is at most the degree of denominator

eg $f(x) = \dfrac{5x^2+3}{x^3+1}$ has $y = 0$ as horizontal asymptote

$f(x) = \dfrac{5x^2+3}{4x^2+1}$ has $y = 5/4$ as horizontal asymptote

$f(x) = \dfrac{5x^2+3}{4x+2}$ has no horizontal asymptote.

**TRICKS**

how to change 0 to ∞ & vice versa: use

$$\lim_{x\to\infty} f(x) = \lim_{x\to 0^+} f(1/x)$$
$$\lim_{x\to-\infty} f(x) = \lim_{x\to 0^-} f(1/x)$$

eg use this to find $\lim_{x\to 0^+} e^{1/x}$ (page 14)

this is a special case of a version of:

$$\lim_{x\to a} f(g(a)) = \lim_{y\to g(a)} f(y)$$
if $g$ is a continuous function at $a$

if faced with "$\infty - \infty$" try to factor. eg $\lim_{x\to\infty} x(x-1)$

$$\lim_{x\to\infty}\left(\frac{x^2-1}{x} - x\right)$$

Note $e^x$ grows faster than any polynomial.