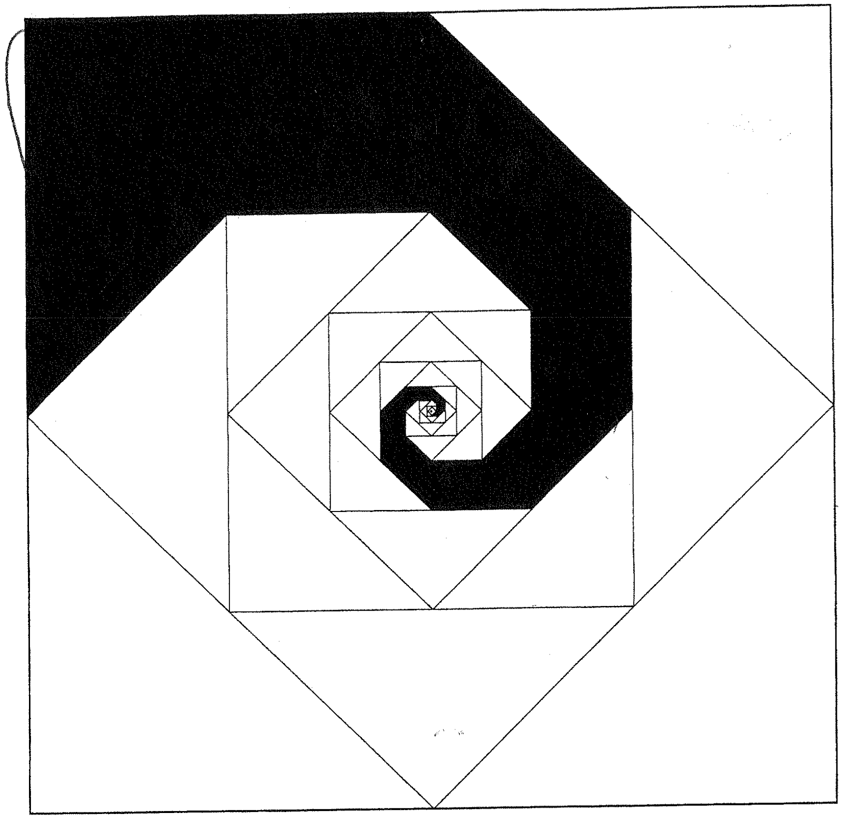
①



This spiral shape is made of infinitely many triangles, spiralling to the center of the square.

What is the area of the spiral shape as a fraction of the area of the square? (You can assume the square has side length one)

What's the answer as a sum?
As a limit?
As a number?

---

②

How do you prove that $\log_{10}(5)$ is irrational?

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③

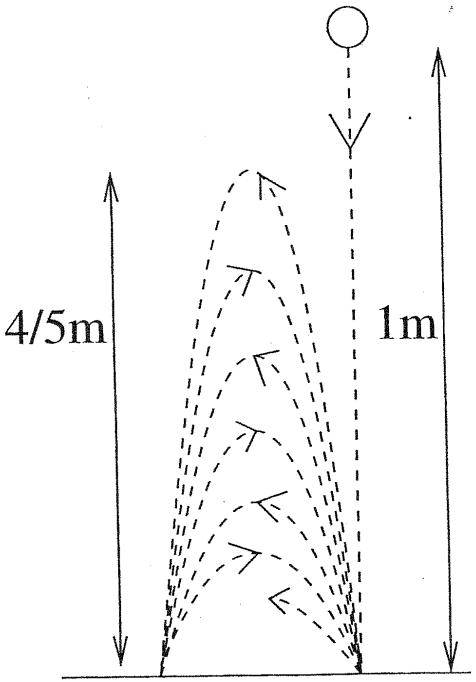
What is the number

$$1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cdots}}}}}}\ ?$$

Does this make sense?
Can this be written as a limit?
If this is a limit of a sequence, what's the sequence, and what's its limit?

4

If a bouncy ball is dropped from a height of 1m, and bounces back up to 4/5 of its previous height on each bounce, how far does it travel in total?



Assume the ball bounces vertically, so you only count vertical distance, which means the two points on the ground in this picture should really be at the same place

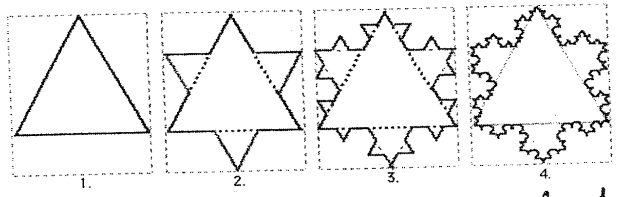
Assume the ball keeps bouncing until it stops. Each bounce is faster than the previous, so it only stops after infinitely many bounces, but made in a finite amount of time. Only 8 bounces are shown in the picture.

What's the answer as a sum?
As a limit?
As a number?

more accurate picture of path of ball - but harder to see what's going on.

5

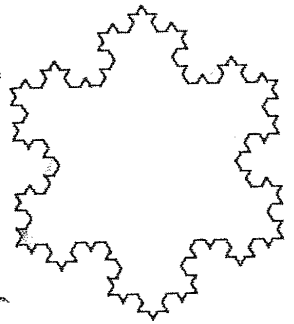
The Snowflake Curve



1. Start with an equilateral triangle whose ~~sides have length~~ area is 1
2. On the middle third of each of the three sides, build an equilateral triangle with sides of length 1/3 of original length. Erase the base of each of the three new triangles.
3. On the middle third of each of the twelve sides, build an equilateral triangle with sides of length 1/9 of original length. Erase the base of each of the twelve new triangles.
4. Repeat the process with this 48-sided figure. See the likeness to a crystal of snow emerge?

At the right, figure 4 is magnified by a power of two.

The "limit curve" defined by repeating this process an infinite number of times, adding more and more, smaller and smaller triangles at each stage, is called the **Koch's SNOWFLAKE CURVE**, named after Niels Fabian Helge von Koch (Sweden, 1870-1924).



What is the area of Koch's snow flake?

what's the answer as a sum?
as a limit?
as a number?