

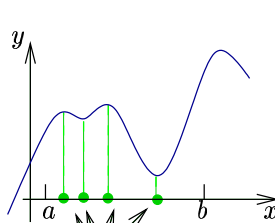
## Optimization Definitions:

- $f(x)$  has an **absolute maximum** at  $x = c$   
if  $f(c) \geq f(x)$  for all  $x$  in the domain of  $f(x)$ .  
In this case  $y = f(c)$  is the **absolute maximum value** of  $f$ .
- $f(x)$  has an **absolute minimum** at  $x = c$   
if  $f(c) \leq f(x)$  for all  $x$  in the domain of  $f(x)$ .  
In this case  $y = f(c)$  is the **absolute minimum value** of  $f$ .
- $f(x)$  has an **absolute maximum** in  $[a, b]$  at  $x = c$   
if  $f(c) \geq f(x)$  for all  $x \in [a, b]$ .  
Then  $y = f(c)$  is the **absolute maximum value of  $f$  on  $[a, b]$** .
- $f(x)$  has an **absolute minimum** in  $[a, b]$  at  $x = c$   
if  $f(c) \leq f(x)$  for all  $x \in [a, b]$ .  
Then  $y = f(c)$  is the **absolute minimum value of  $f$  on  $[a, b]$** .
- $f(x)$  has a **local maximum** in  $[a, b]$  at  $x = c$   
if  $f(c) \geq f(x)$  for all  $x$  close to  $c$ .
- $f(x)$  has a **local minimum** in  $[a, b]$  at  $x = c$   
if  $f(c) \leq f(x)$  for all  $x$  close to  $c$ .
- $f(x)$  has a **critical value** at  $x = c$   
if  $f'(c) = 0$  or if  $f'(c)$  does not exist.

## Strategy for finding max and min of $f(x)$ on $[a, b]$ :

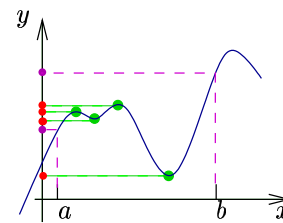
1. Locate the critical numbers of  $f(x)$  in  $[a, b]$
2. Find  $f(x)$  at the critical numbers
3. Find  $f(a)$  and  $f(b)$
- 4A. The absolute maximum of  $f(x)$  in  $[a, b]$  is the largest of the values in steps 1 and 2.
- 4B. The absolute minimum of  $f(x)$  in  $[a, b]$  is the smallest of the values in steps 1 and 2.

Step 1:  
finding  $x$  values

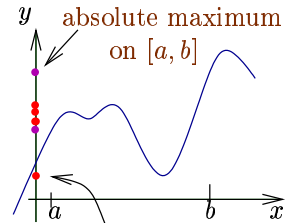


Critical numbers in  $[a, b]$

Step 2 and 3:  
finding  $y$  values



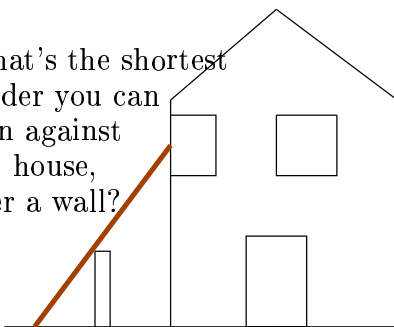
Step 4:  
finding max and min  $y$  values



absolute minimum on  $[a, b]$

what's the biggest rectangle that fits in a circle?

What's the shortest ladder you can lean against the house, over a wall?



What's the largest area you can enclose by a 28 yard long fence?

## Strategy for optimization word problems:

1. **Read** the question.
2. Draw a **diagram** showing all quantities involved.
3. Work out what quantity has to be maximized. Give this a name, e.g., call this  $Q$ .
4. Work out what this quantity depends on, and name these, e.g., say  $Q$  depends on an  $x$  and  $y$ , so  $Q = Q(x, y)$ .
5. Work out what the constraints to the problem are, e.g., say  $y = f(x)$
6. Write  $Q$  as a function of one variable, e.g., by substitution  $Q = Q(x, f(x))$
7. Check whether  $x$  must be an interval  $[a, b]$ .
8. Apply the strategy for finding max or min values of a function on an interval, as above.
9. The numbers you get from step 7 should be interpreted in terms of the original problem.