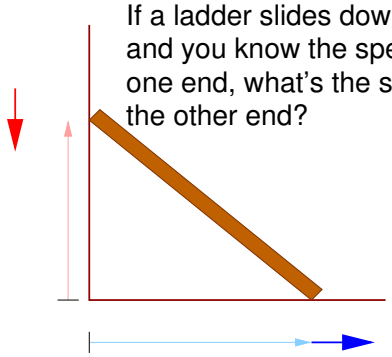
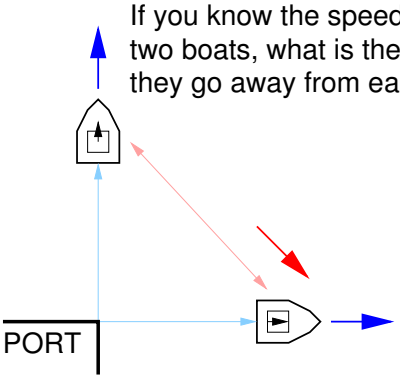
If a ladder slides down a wall, and you know the speed of one end, what's the speed of the other end?



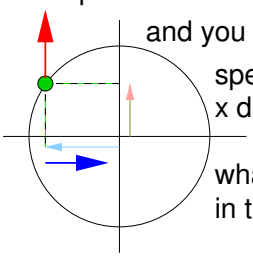
If you know the speed of two boats, what is the speed they go away from each other?




# RELATED RATES

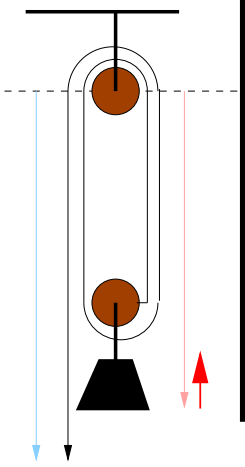
## Typical problems

If a point moves on a circle, and you know its speed in the x direction, what is the speed in the y direction?



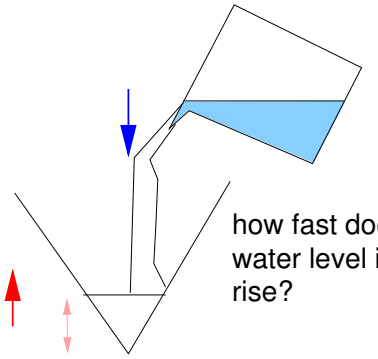
Method to solve related rates problems:

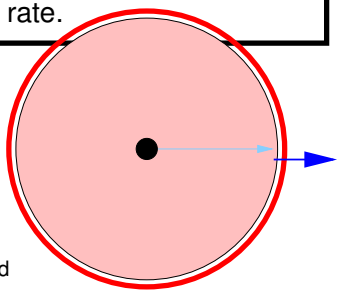
1. Read the question carefully.
2. Draw a diagram.
3. Work out what is changing, introduce notation, and label diagrams, including all relevant quantities
4. Write the rates of change in the question in terms of derivatives
5. Find a relation between the quantities involved
6. Differentiate the relationship to find how the rates of change are related.
7. Use substitution of known rates and quantities to find the unknown rate.



If you pull down on the rope from a pulley at 5 foot per second, at what speed does the weight rise?

If water pours into the cone at 1 gallon per minute, how fast does the water level in the cone rise?





If you know how fast the radius of a circle is increasing, how fast if the area increasing?

In this picture, red arrows indicate rates of change you need to find
blue arrows indicate rates of change you know
pale blue and pale red indicates the quantities that are changing.
You need to relate the pale blue and pale red quantities, and use this to relate the rates of change you're interested in

RELATED RATES

# MEAN VALUE THEOREM

MVT says there is a tangent with the same slope as the secant

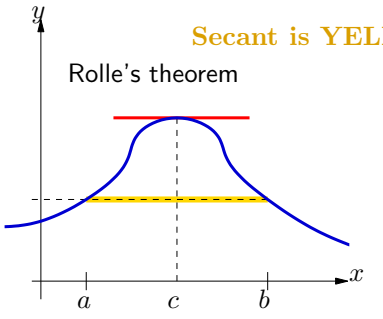
Secant is YELLOW  Tangent is RED

### Rolle's Theorem

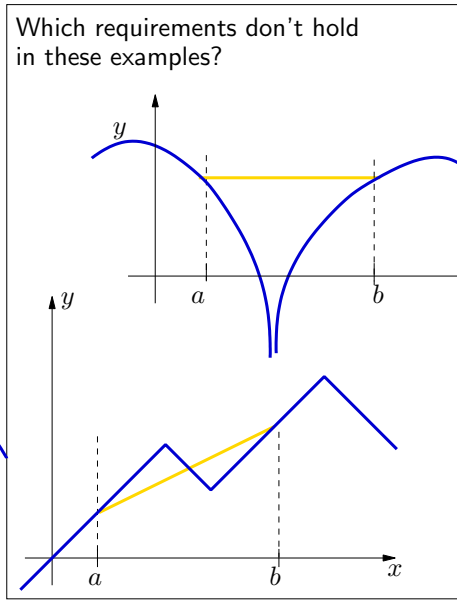
Let $f$ be a function that satisfying:

**1.** $f$ is continuous on the closed interval $[a, b]$.
**2.** $f$ is differentiable on the open interval $(a, b)$.
**3.** $f(a) = f(b)$

Then there is a number $c$ in $(a, b)$ such that $f'(c) = 0$.




Which requirements don't hold in these examples?




### The Mean Value Theorem

Let $f$ be a function satisfying:

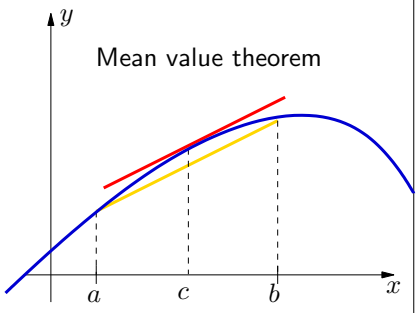
**1.** $f$ is continuous on the closed interval $[a, b]$.
**2.** $f$ is differentiable on the open interval $(a, b)$.

Then there is a number $c$ in $(a, b)$ with

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

This can also be written as:

$$f(b) - f(a) = f'(c)(b - a)$$




(Handout for Math 1550, section 5 (Dr. H. Verrill). For October 20 and 21, 2004.)