LIMITS

What is it? (2.1, 2.4)

$$\lim_{x \to a} f(a) = L$$

Roughly: as x gets closer to a, $f(x)$ gets closer to L.
Tells us what value a function "should have" at a (if you can't just plug in a), or what value it's getting closer to (e.g., when a becomes infinite).

Quiz: What are left and right limit?

How to find it

Use a **table** (2.2) or **limit laws and algebra** (2.3)

from the left:

x	$\frac{\sqrt{(5-x)}-2}{x-1}$
0.5	−0.242...
0.9	−0.248...
0.99	−0.249...

from the right:

x	$\frac{\sqrt{5-x}-2}{x-1}$
1.5	−0.258...
1.1	−0.251...
1.01	−0.250...

The blah of the limit is the limit of the blah

Where "blah" could be product, sum, difference, power, root, quotient (provided you don't divide by 0), etc.

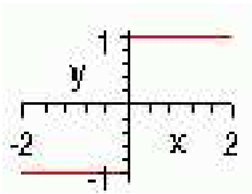
E.g., $\lim_{x\to 1} \frac{\sqrt{5-x}-2}{x-1} = \lim_{x\to 1} \left(\frac{(\sqrt{5-x}-2)(\sqrt{5-x}+2)}{(x-1)(\sqrt{5-x}+2)} \right) = \lim_{x\to 1} \left(\frac{(5-x)-4}{(x-1)(\sqrt{5-x}+2)} \right) = \lim_{x\to 1} \left(\frac{(1-x)}{(x-1)(\sqrt{5-x}+2)} \right) = \lim_{x\to 1} \left(\frac{-1}{(\sqrt{5-x}+2)} \right) == -1/4$

** To get the hang of this you *must* do lots of examples on your own! See 2.3 exercises **

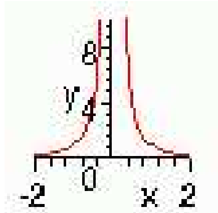
What does it tell us?

Discontinuity

Examples when the limit doesn't exist:

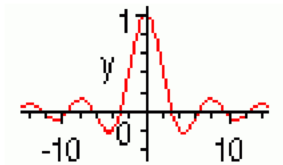


$|x|/x$ at 0
jump discontinuity
(left and right)
limit do exist)

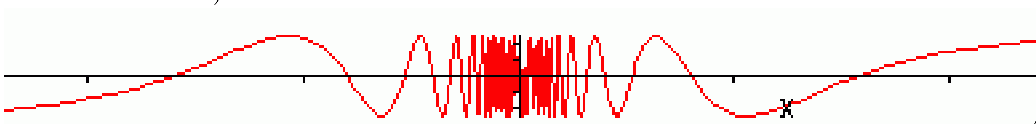


$1/x^2$ at 0
infinite discontinuity

Or does, though $f(a)$ doesn't



$\sin(x)/x$ at 0
removable discontinuity



$\sin(1/x)$ at 0 — "wild behaviour"

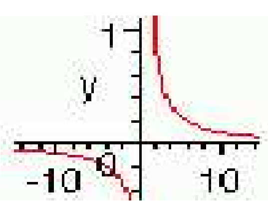
"opposites"

Asymptotes

Vertical

When does $f(x) \to \infty$?
(explosions; points to avoid)

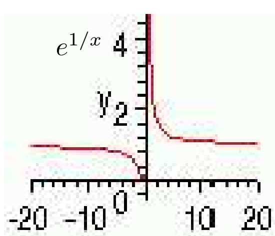
$1/(x-1)$



Asymptotes: $y = 0$ and $x = 1$

Horizontal

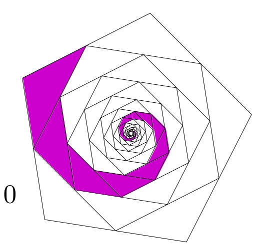
What happens when $x \to \infty$?
(long term trends)



Asymptotes: $y = 1$ and $x = 0$

$0.99999.... = 1$

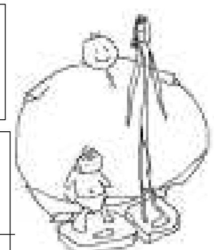
Quiz: In what sense is a real number a limit? How is a spiral a limit?



Continuity (2.5)

$f(x)$ is continuous at a if
$$\lim_{x\to a} f(x) = f(a).$$
All quantities must exist!

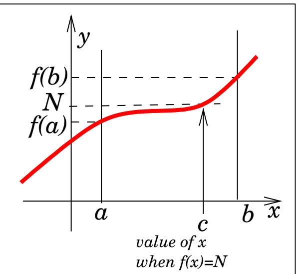
Quiz: What do left and right continuity mean? What is continuity on an interval?



Did you ever confuse your height with your weight?

Intermediate Value Theorem

IVT:
If $f(x)$ is **continuous** on $[a, b]$,
and if N is between $f(a)$ and $f(b)$,
then $f(c) = N$ for some $c \in [a, b]$.



E.g.,
I know I was once three feet tall
— growth is continuous.
But I can't say whether the score was once 25 points,
since baseball scores are not.

Variation on the intermediate value theorem:
If $f(x)$ and $g(x)$ are both continuous on $[a, b]$,
and $f(a) > g(a)$ and $g(b) > f(b)$,
then somewhere between a and b, f and g are equal.
E.g., my height in inches once equalled my weight in pounds.