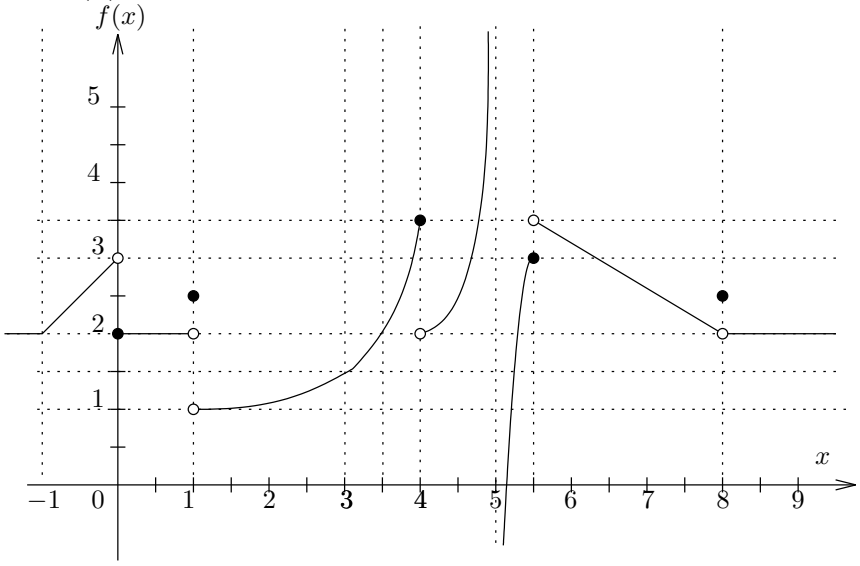
SAMPLE FIRST TEST QUESTIONS :

Calculus 1550, Section 5, Fall 2004.

[In an actual test, more space would be given for writing answers.]

This is a list of possible questions; a test would not be this long.

Q1. A function $f(x)$ has domain $\mathbb{R}$, and is given by the following graph for $x \in [-1, 9]$, and by $f(x) = 2$ otherwise.



i. What are all the discontinuities of $f(x)$?

ii. Complete the following table:

a	$f(a)$	$\lim_{x\to a^-} f(a)$	$\lim_{x\to a^+} f(a)$	$\lim_{x\to a} f(a)$	right continuous at a?	left continuous at a?
-1						
4						
8						

Q2. Evaluate the following limits, using limit laws and techniques for computing limits exactly. Show your working, or explain your reasoning, but it is not necessary to write down which laws you are using. If a limit does not exist, explain why not.

i. $\lim_{x\to 2} \sqrt{x^2 + x + 3}$

ii. $\lim_{x\to 2} \dfrac{x^3 - 8}{x^2 - 4}$

iii. $\lim_{x\to 0} \dfrac{(3+x)^2 - 9}{x}$

iv. $\lim_{x\to 0} \dfrac{|x|}{x}$

v. $\lim_{x\to 1} \dfrac{\sqrt{5-x} - 2}{x - 1}$

Q3. Given some numbers a and b, define a function

$$f_{ab}(x) = \frac{x^3 + ax + b}{x^2 + 4x}.$$

Fill in the blanks in the table, to give values of a and b to make the last column correct. Write one number in each space.

a	b	$\lim_{x\to 0} f_{ab}(x)$
	1	undefined
		2
		0

Q4. In this question, you do not need to actually find the limits, but you must write down exactly which rules you would use to find the limit. You must write the shortest possible list of rules. If you write every rule, you will receive no points.

The list of rules and operations are:

Limit laws

1. The limit of a sum is the sum of the limits
2. The limit of a difference is the difference of the limits
3. The limit of a constant times a function is the constant times the limit of the function
4. The limit of a product is the product of the limits
5. The limit of a quotient is the quotient of the limits
6. The limit of a root is the root of the limit

Special cases of limits

7. The limit of a constant is the constant
8. The limit of the function $f(x) = x$ as x approaches a is a

Algebraic operations

9. factorisation
10. cancellation of common factors

Write the numbers of the rules used in the right hand column.

	limit to find	rules you would use to find the limit
i.	$\lim_{x\to 2} \frac{x^2 - 2}{x^3 - 2}$	
ii.	$\lim_{x\to 2} \frac{x^3 - 8}{x^2 - 10x + 16}$	
iii.	$\lim_{x\to 0} \frac{(x^2 - 2)^2 - x^4}{x^2}$	
iv.	$\lim_{x\to 0} \frac{\sqrt{x^2 + 3x}}{x}$	
v.	$\lim_{x\to 1} \frac{\lvert x^2 - 1\rvert}{x - 1}$	

Q5. A function $f(x)$ is defined by

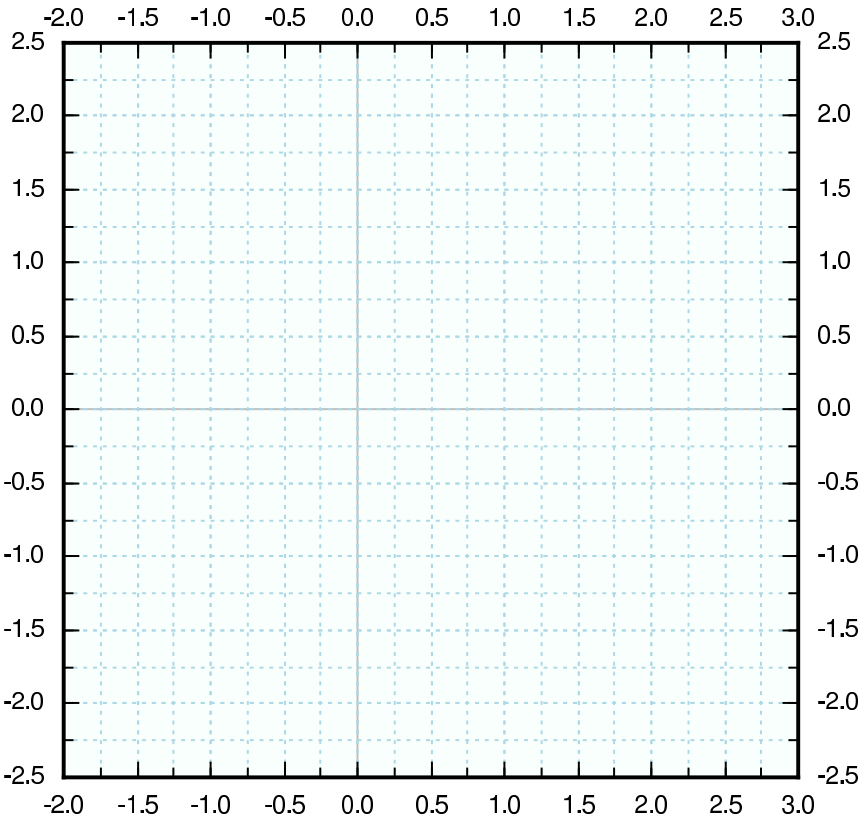
$$f(x) = \begin{cases} 0 & \text{if } x < -1 \\ -x & \text{if } x \in [-1, 0) \\ x^2 & \text{if } x \in (0, 1) \\ x & \text{if } x \in [1, 2] \\ -1 & \text{if } x > 2 \end{cases}$$

i.

a) Where is $f(x)$ discontinuous?

b) Where is $f(x)$ not defined?

ii. Sketch the graph of $f(x)$ on the following grid:



iii. [15] Complete the following table:

a	$f(a)$	$\lim_{x \to a^-} f(a)$	$\lim_{x \to a^+} f(a)$	$\lim_{x \to a} f(a)$	right continuous at a?	left continuous at a?
-1						
0						
2						

Q6.

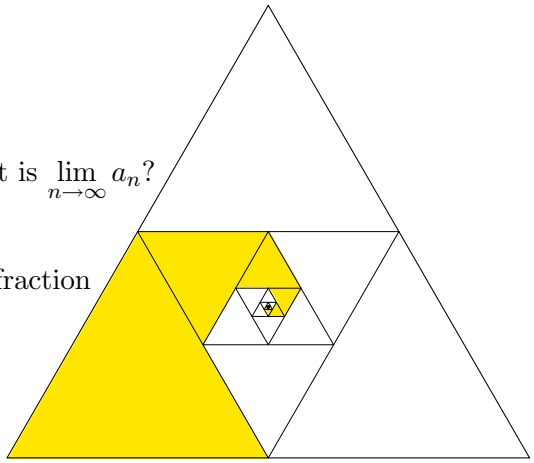
1. What is the mathematical definition of continuity?
2. On what intervals is the function $f(x) = \frac{\sin(x)}{x(x-1)}$ continuous? Justify your answer.
3. Where is $f(x) = \frac{\sin(x)}{x(x-1)}$ discontinuous? Justify your answer.
4. For each discontinuity of $f(x) = \frac{\sin(x)}{x(x-1)}$, state whether it is a removable discontinuity, infinite discontinuity, or jump discontinuity, or another kind of discontinuity.

Q7.

1. Write down the mathematical definition of the limit of a function $f(x)$ as x approaches a from the right. Write your answer in complete grammatical sentences, and include a description of the mathematical symbols usually used to denote this limit.
2. Give an example of a function with different left and right limits at some point.
3. Give an example of a function where the limit of $f(x)$ as x approaches a exists, but $f(a)$ does not exist.
4. State the limit law known as the "quotient rule", and give an example of where you can use it to find a limit.

Q8.

1. If $f(x) = \dfrac{2^x - 1}{2^{x+3}}$, what is $\lim\limits_{x\to\infty} f(x)$?
2. If $a_1 = 0.9$, $a_2 = 0.99$, and so on, so that $a_n = \sum\limits_{i=1}^{n} \dfrac{9}{10^n}$, then what is $\lim\limits_{n\to\infty} a_n$?
3. What is the shaded spiral area in the diagram on the right, as a fraction of the area of the large triangle?
   How would you write this as a sum of infinitely many terms?



Q9.

In the following questions, state whether the situation is
(a) possible, but not certain, (b) definitely has happened, or (c) is impossible.

In case (a), give examples to explain why it may or may not have happened.

In case (b), justify your answer, say what results you are using, and what functions you work with. Also give an idea of how many times this has happened, e.g., just once, or many times?

In case (c), explain why it is impossible

1. Has there ever been a moment when the temperature in Sydney, Australia, is exactly the same as the temperature in Baton rouge?
2. A seed, which will grow into a pine tree, sprouts its first shoot on the same day as a baby is born. 20 years later, both are living. Were they ever exactly the same height?
3. In 1990, Mr. X was a millionaire, whilst Mr. Y was deeply in debt. Today Mr. X has lost all his fortune and is bankrupt, while Mr. Y has just made his first million. Was there ever a moment that they had exactly the same amounts of money in their bank accounts?