Calculus 1550, section 5. Monday 20 September, 2004. First test.

STUDENT NAME:

This test paper has 4 questions. Points for questions are given in square brackets.
The total is 50.

Write your **name** on this page, and your **initials** on each sheet.

If there is not enough space on the printed side of each sheet, you should also use the reverse, but please indicate on the front if you continue your answer on the back of the page.

Q1. [5 points]
What does it mean (in terms of limits) for a function $f(x)$ to be continuous at a point a?

Q2. [15 points]

A. Give examples of

1) a function $f(x)$ with an infinite discontinuity at $x = 2$.
2) a function $g(x)$ with a jump discontinuity at $x = 2$.
3) a function $h(x)$ with a removable discontinuity at $x = 2$.

B. Sketch graphs of each of your functions $f(x), g(x), h(x)$, making clear which is which, and showing the discontinuity clearly.

C. For your functions f, g and h as in part A, fill in the following table. If the limit does not exist, you can write "D.N.E". If the limit is $+\infty$ or $-\infty$, you should write this rather than "D.N.E", to rceeive full points. If the limit is a number, write the number.

dicontinuity	left limit	right limit	limit
infinite	$\lim_{x\to 2^-} f(x) =$	$\lim_{x\to 2^+} f(x) =$	$\lim_{x\to 2} f(x) =$
jump	$\lim_{x\to 2^-} g(x) =$	$\lim_{x\to 2^+} g(x) =$	$\lim_{x\to 2} g(x) =$
removable	$\lim_{x\to 2^-} h(x) =$	$\lim_{x\to 2^+} h(x) =$	$\lim_{x\to 2} h(x) =$

Q3. [20 points]

A. Find the value of the following limit, giving **every** step of working, Write which limit laws you use at every step, (e.g., write "sum law" or "product law", etc, and state if you factor or cancel factors, by writing "cancel" or "factor".)

$$\lim_{x\to 2}\left(\frac{x^2-4}{x-2}\right)$$

B. Find the value of the following limit. Show your working, or give your reasoning, but you do **not** need to write in words what you are doing, or say which limit laws you are using.

$$\lim_{x\to 4}\left(\frac{x-4}{\sqrt{x}-2}\right)$$

Q4. [10 points]

I want to cut off the corners of a square with one straight cut for each corner, so that the shape left over is a regular octagon. The picture below shows what is meant by the four cuts, though the result in this case is not regular. (A regular octagon has 8 sides, and each side has the same length, and the angles are all 45 degrees).

Explain how the intermediate value theorem implies that I know there is a way to do this, i.e., I know that this is not an impossible task, even if I do not yet know exactly what the solution is.

(You can receive partial credit if instead you find how to solve this problem precisely, without using IVT; **or** if you state the intermediate value theorem, and say something about how it is used in general, **or** if you give any kind of "common sense" argument about why this should be possible).