

STUDENT NAME:

Calculus 1550, section 5. Second test. Tuesday, October 3, 2004.

This test paper has 4 pages. Points per question are given in square brackets.

The total is 50 points.

Put your name on this sheet and your initials on each page.

Q1. [5 points]

What is the definition of the derivative of a function $f(x)$ in terms of limits? write the definition as a mathematical formula, and sketch a labelled diagram showing the geometrical meaning of this definition.

Q2. [5 points]

Give an example of a function $f(x)$ which is continuous at $x = 5$, with $f(5) = 2$, and such that $f'(5)$ is not defined. Give the equations for $f(x)$ and for $f'(x)$, and sketch graphs of $f(x)$ and of $f'(x)$.

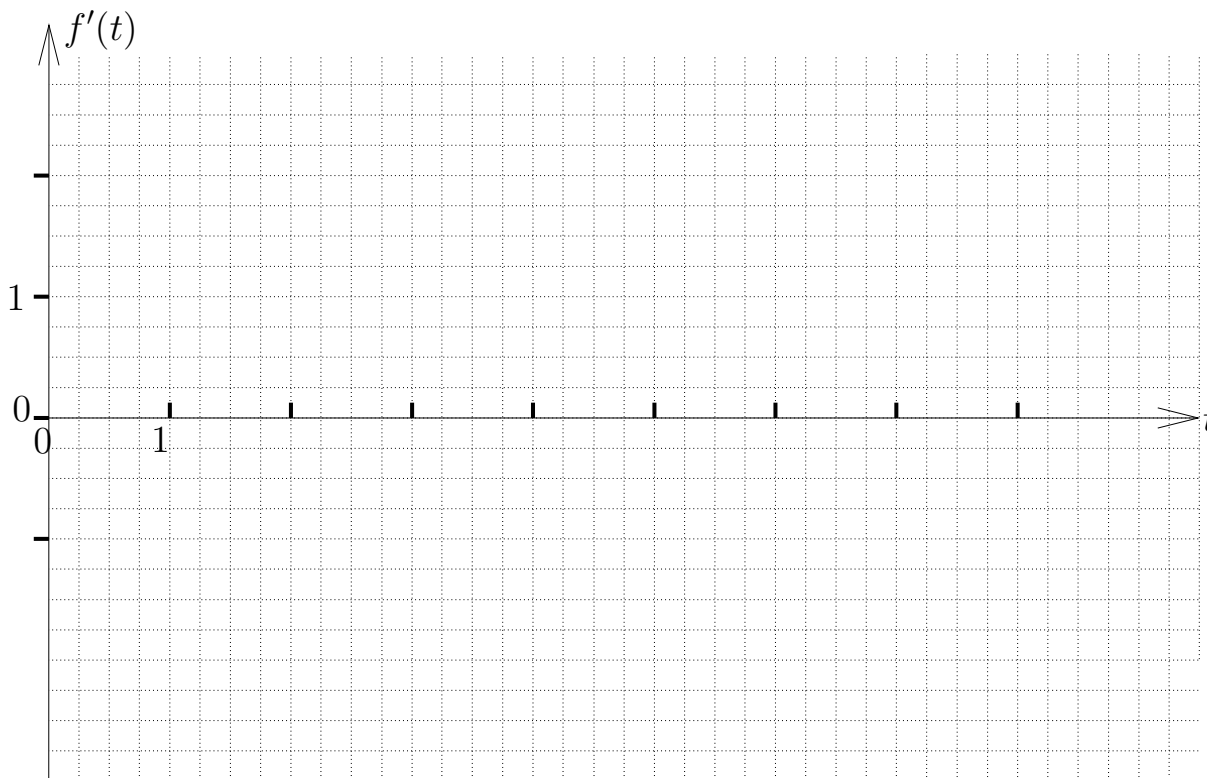
Q3. [10 points]

$f(t)$ is a function with a graph as follows:



Sketch the graph of $f'(t)$ below. Make sure your graph shows the following features:

- i Where $f'(t)$ is zero, positive and negative.
- ii Where $f'(t)$ is decreasing or increasing.
- iii Where $f'(t)$ is undefined, infinite (by drawing an asymptote), or has a discontinuity.
- iv Exact values of $f'(t)$ when $f'(t)$ is constant on some interval.



Q4. [10 points]

For which values of x does the graph of the following function have a horizontal tangent?

$$f(x) = x^3 - 3x^2 + 2$$

Sketch a graph of the function showing these horizontal tangent lines.

Q5. [10 points.] Find the equation for the tangent of the following function at $(1, -5)$

$$f(x) = x^2 - x - 5$$

Sketch a graph of the function and this tangent line.

Q6.

Find the derivatives of the following functions [5 points each].

i. $f(x) = (x^3 + 2) \sin(x)$

ii. $f(x) = \frac{x^3 + x + 1}{\cos(x)}$