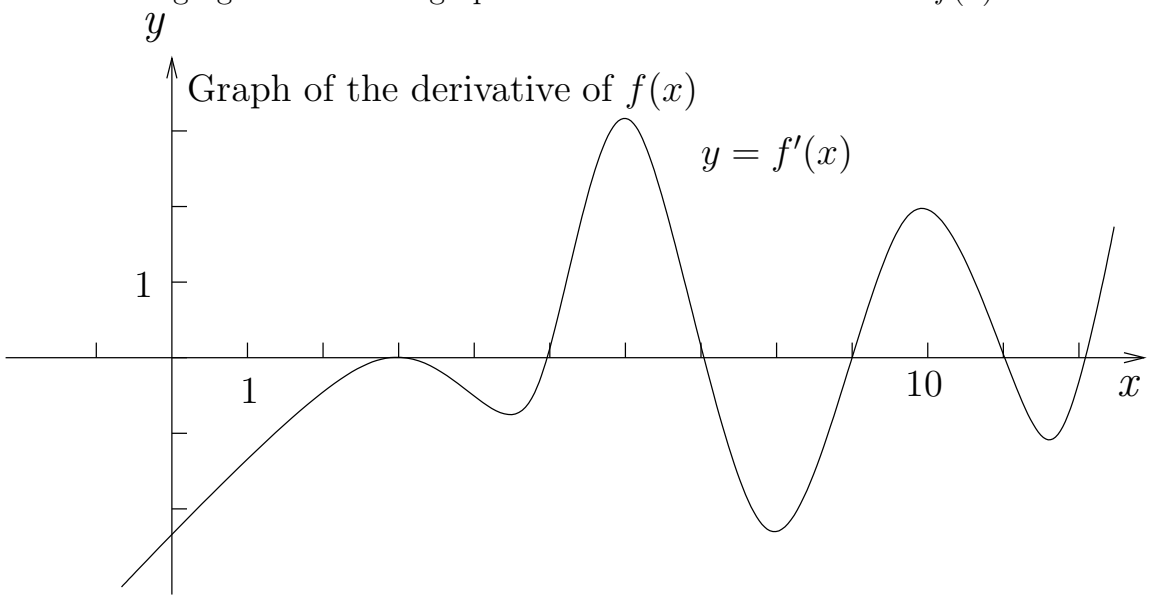
Calculus 1550, section 5. Thursday 28 October, 2004. Third test.

STUDENT NAME:

This test paper has 4 questions. Points for questions are given in square brackets.
The total is 50.
Write your **name** on this page, and your **initials** on each sheet.

If there is not enough space on the printed side of each sheet, you should also use the reverse, but please indicate on the front if you continue your answer on the back of the page.

Q1. [5 points]
The following figure shows the graph of the derivative of a function $f(x)$



1. On which intervals with $1 \leq x \leq 10$ is $f(x)$ increasing?

2. For which value of c in $[1, 10]$ does $f(x)$ have a local maximum at $x = c$?

3. Does $f(x)$ have any inflection points? If so, where are they?

Q2.

A. [4 points] Find the derivative of

$$f(x) = (\sin(x^2 + 3))^5$$

B. [4 points] Find the derivative of

$$f(x) = \frac{\ln(x^2)}{x}$$

C. [5 points] Given that

$$y^3 + 2y = x^6 + x^2$$

find $\frac{dy}{dx}$ as a function of x and y.

Q3.

A 4 meter long ladder ladder leans against a wall.

At time $t = 0$ the ladder is vertically against the wall.

The top end of the ladder starts to move so that its distance from the ground at time t is

$$s(t) = (4 - t^2) \text{ m} \qquad \text{for} \quad 0 \leq t \leq 2$$

At time $t = 2$ the top end of the ladder reaches the ground and then the ladder stops moving.

A. [1 point] What is the speed of the top end of the ladder at time t?

B. [10 points] Find a formula for the speed of the bottom end of the ladder. (This can be given as a formula in terms of t and the distances of the ends of the ladder from the point where the wall and ground meet.)

C.[1 point] What is the speed of the bottom end of the ladder when $t = 2s$?

Q4. [20 points]

A particle moves in a straight line, so that its distance in feet from its starting point at time t is given by

$$s(t) = 2t + t^2 - t^3$$

where t is measured in seconds.

A. What is the velocity of the particle at time t?

B. How far does the particle travel from $t = 0$ to $t = 1$?

C. What is the average velocity from $t = 0$ to $t = 1$?

D. At what times between $t = 0$ and $t = 1$ is the instantanous velocity equal to the average velocity from $t = 0$ to $t = 1$?

E. What is the maximum value of the velocity from $t = 0$ to $t = 1$?