11 + 3 + 10 + 6 = 30 points total.

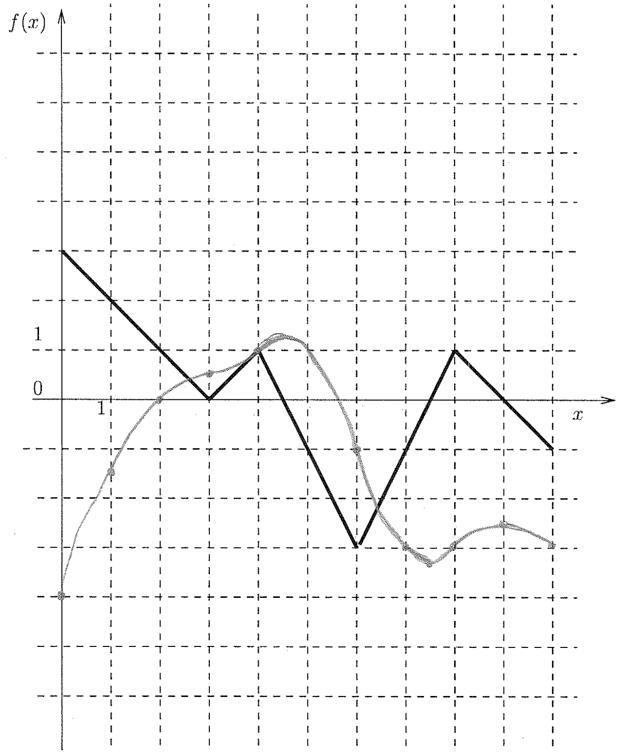
STUDENT NAME:

Calculus 1550, section 5. test 4. November 23 2004

1) graded out of 11

1. Below the graph of a function $f(x)$ is sketched. Another function $g(x)$ is defined by

$$g(x) = \int_2^x f(t)dt$$



1. On what intervals is $g(x)$ increasing?

(0,3), (3,4.5), (7.5,9)

2. For which value of x is $(x, g(x))$ a local minimum of g(x)?

7.5

3. For which value of x is $(x, g(x))$ a local maximum of g(x)?

4.5, 9

4. Does $g(x)$ have any inflection points? If so where are they?

yes; at $x = 3, 4, 6, 8$

5. Where is $g(x)$ concave up?

(3,4) , (6,8)

6. On the same grid, sketch a graph of $g(x)$, showing features from parts 1-5 of this question.

7. What is the maximum value of $g(x)$ on $[0, 10]$?

1.25 at $x = 4.5$

8. What is the minimum value of $g(x)$ on $[0, 10]$?

-4 at $x = 0$

9. What is $g'(7)$?

-1

10. What is $g''(7)$?

2

11. If $\int_a^x f(t)\mathrm{d}t = g(x) - 1$, what is a possible value of a?

4 (or 5)

2. If graded out of 3

$$\sum_{i=1}^{n} \frac{5i\sin\left(\frac{i}{n}\right)}{n^2}$$

is a Riemann sum approximation of the integral

$$\int_a^b f(x)dx$$

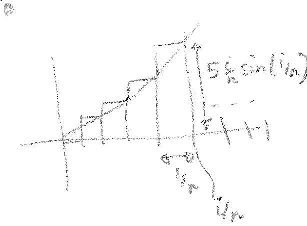
what are a and b and $f(x)$?

let $\boxed{a = 0 \quad \& \quad b = 1 \quad , \& \quad f(x) = 5x\sin(x)}$

divide [0,1] into n intervals, each width 1/n

take right Riemann sum. then

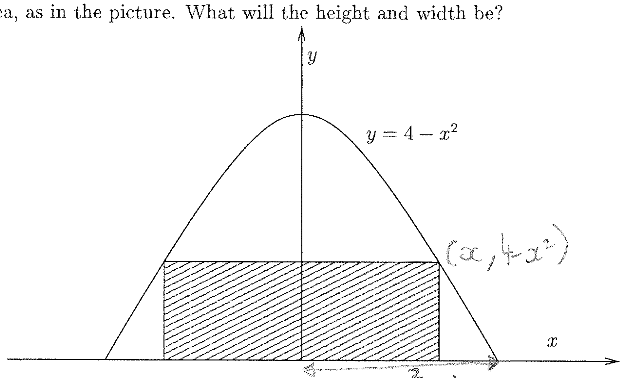
$$\int_0^1 5x\sin(x)\,dx \approx \sum_{i=1}^{n} \underbrace{5\left(\frac{i}{n}\right)\sin\left(\frac{i}{n}\right)}_{\text{height of strip}} \times \underset{\substack{\uparrow \\ \Delta x \\ \text{width of strip}}}{\frac{1}{n}}$$



(other answers also possible)

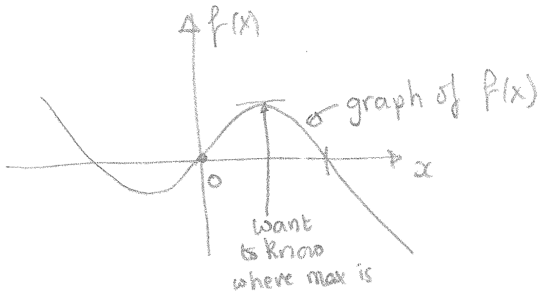
(10 points)

3. My new tent design has crossection given by an equation $y = 4 - x^2$, as shown in the diagram. I want to make a rectangular door with the largest possible area, as in the picture. What will the height and width be?



area of door $= 2x(4-x^2) = 2x(2-x)(2+x)$

$= f(x) = 8x - 2x^3$



note $f(2) = 0$

so must take $x \in [0,2]$

at max value, $f'(x) = 0$.

$f'(x) = 8 - 6x^2$ when $f'(x)=0$, $8 = 6x^2$

$4/3 = x^2 \Rightarrow x = \pm 2/\sqrt{3}$

since must have $x \in [0,2]$, use $2/\sqrt{3}$

$f(0)=0$, $f(2) = 0$, $f(2/\sqrt{3}) = \frac{2}{\sqrt{3}}(4 - 4/3) = 2/\sqrt{3} \cdot 8/3 = 16/3\sqrt{3}$

$\boxed{\begin{array}{l} \text{height} = 4 - 4/3 = 2\,2/3 \\ \text{width} = 4/\sqrt{3} \quad (=2x) \end{array}}$

with these values, get max area of door, $16/3\sqrt{3}$ units^2

(2 pts)

3. Find the most general form of the following indefinite integral

$\int \frac{x^4}{\sqrt{x^5+3}}dx$

let $u = x^5 + 3$

$du = 5x^4\,dx$

$\int \frac{x^4 dx}{\sqrt{x^5+3}} = \int \frac{1/5\, du}{\sqrt{u}} = \int \frac{u^{-1/2}}{5} du = \frac{2}{5}u^{1/2} + C$

$= 2/5\sqrt{x^5+3} + C$

4. (2 pts)

If $f(x) = \int_0^{x^2} \frac{3t+\sin(t)}{t} dt$, what is $f'(x)$?

$f'(x) = 2x\left(\frac{3x^2 + \sin(x^2)}{x^2}\right)$

(chain rule)

6. (2 pts)

Use L'hopital's rule to compute the limit

$$\lim_{x \to 0} \frac{\sin(x^2)}{\sin(x)^2}$$

$= \lim_{x\to 0} \frac{2x\cos(x^2)}{2\sin(x)\cos(x)}$ ← used L'hopitals rule

$= \lim_{x\to 0} \frac{x}{\sin(x)} \lim_{x\to 0} \frac{\cos(x^2)}{\cos(x)}$ ← use limit of products is product of limits

$= \lim_{x\to 0} \frac{1}{\cos(x)} \times \frac{1}{1}$ ← used L'hopital on 1st limit, pluged in on 2nd

$= \frac{1}{1} \times 1 = 1$ ← plug in, since limit of numerator & denomat $\neq 0$, (& function are continuous)