Solving discrete problems — Math 2020, Spring 2005

Schedule For Tuesday, February 1, 2005.

1. Venn diagrams (pages 60–67) and Euler diagrams (pages 100–101), and connectives $\neg, \vee, \wedge, \rightarrow, \leftrightarrow$.
   ∗ Exercise on this part of class: Draw Euler diagrams for all theorems
   ∗ we've proved so far, and all in this lecture. In each diagram, you should include a set
   ∗ for each hypothesis, and a set for the conclusion.
2. Easy proof by contradition : if neither of a and b is zero, then $(a+b)^2 \neq a^2 + b^2$.
   ∗ Exercise on this part of class: Read up on proof, especially proof by contradiction, e.g., see:
   ∗ http://zimmer.csufresno.edu/ larryc/proofs/proofs.contradict.html (by L Cusick)
   ∗ http://www.math.gatech.edu/ heil/handouts/proofs.pdf (by C. Heil)
3. Discussion on distribution of primes.
4. Some important proofs in number theory:
   i. Proof that there are infinitely many primes.
      ∗ Page 125 in the course book. If you don't have the book, see:
      ∗ http://odin.mdacc.tmc.edu/∼krc/numbers/infitude.html

Break

5. Continuing important proofs in number theory:
   ii. Results about the GCD (greatest common divisor).
      ∗ (see page 121–123 or next reference below,)
   iii. The Euclidian algorithm, and proof it works.
      ∗ (see page 275 or http://www.maths.monash.edu.au/mth3122/a4lect2.pdf)
   iv. Unique prime factorisation in the integers:
      ∗ (see page 126 or http://www.maths.monash.edu.au/mth3122/a4lect3.pdf

Learning to read and understand proofs

Most (but not all) theorems have the form

$$\{\text{hypothesis}\} \rightarrow \{\text{conculsion}\}$$

Many proofs include the following:

i. Use the definitions of terms used in stating the hypothesis.
ii. Use previously proved theorems.
iii. Mathematical operations allowed in the system being considered.
iv. Use logical arguments, e.g., proof by contradiction.

You should learn to recognise all of the above, and be able to answer the list of questions below for any given proof.

Example problem for quiz on Thursday:
Read the following proof:

> **Theorem:** Let a be an integer. Then if a is even, and if a is the difference of the squares of two integers, then a must be divisible by 4.
> **Proof:** Suppose $2|a$, and $a = c^2 - d^2$, where n, c, d are integers.
> Then $a = (c-d)(c+d)$. Since $2|a$, then $2|(c-d)$ or $2|(c+d)$. There are two cases to consider.
> In the first case, if $2|(c-d)$, then $c - d = 2m$ for some integer m, and so $c + d = c - d + 2d = 2m + 2d = 2(m+d)$, so $2|(c+d)$.
> In the second case, if $2|(c+d)$, then $c + d = 2n$ for some integer n, and so $c - d = c + d - 2d = 2n - 2d = 2(n-d)$, so $2|(c-d)$.
> In either case, we find that 2 divides $c+d$ and $c-d$, so if we have $c - d = 2n$ and $c + d = 2m$ for integers m and n, then $a = 2m2n = 4mn$, so a is divisible by 4. **QED**.

Questions.

1. What are the hypthesis of this theorem? List each one separately.
2. What is the conclusion of this theorem?
3. Write a list of what definitions are used in this theorem.
4. Circle where each hypothesis is used in this theorem; make sure to include every hypothesis listed in (1.)
5. Write a list of all the previously proved (or assumed) results and theorems which are used in this proof.
6. For each hypothesis, find an example where that hypothesis is false, but the other hypothesis are true, and the conclusion is false.
7. Are there any examples where the hypothesis are not all true, but the conclusion still holds? If so, give an example.
8. Give an example where all the hypothesis are true, and say what the conclusion says in this case.
9. Draw an Euler diagram for this proof, including circles for each of the hypothesis and conclusion, and showing the examples you gave for parts
10. Is this a proof by contradiction? Is this a direct proof?
    (Later you should also be able to say whether other proof techniques are used, such as proof by induction.)

Appologies: I will not be in my office during usual office hours on Wednesday February 2.