Reveiw of Arithmetic

A prime is a positive integer which is only divisible by itself & 1 (usually 1 is not considered to be prime)

ex: 2, 3, 5, 7, 11, 13, 17, 19

if # is not prime, it is called composite

ex: $4 = 2 \cdot 2$, $6 = 2 \cdot 3$, $8 = 2 \cdot 2 \cdot 2$

Prime factorization of positive integer:

m is an expression of the form $m = p_1 \times p_2 \times \ldots \times p_s$

$m = p_1^{a_1} \times p_2^{a_2} \times \ldots \times p_s^{a_s}$

p_i are primes, a_i are positive integers

ex: $15 = 3 \cdot 5$, $90 = 3 \times 3 \times 2 \times 5 = 3^2 \times 2 \times 5$

A factor (divisor) of a positive integer m is a positive integer a such that there is some positive integer b with $m = ab$

Exercise 1: # of divisors of 15 is $\#\{1, 3, 5, 15\} = 4$

2: # of divisors of 30 $\#\{1, 2, 3, 5, 6, 10, 15, 30\} = 8 \Rightarrow 2 \cdot 3 \cdot 5$

90 $\#\{1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90\} = 12 \Rightarrow 2 \cdot 3 \cdot 3 \cdot 5$

64 $\#\{1, 2, 4, 8, 16, 32, 64\} = 7 \Rightarrow 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

123456 $\#\{1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 64, 96, 192, 643, 1286, 1929, 2572, 3858, 5144, 7716, 10288, 15432, 20576, 30864, 41152, 61728, 123456\} = 28$

$\Rightarrow 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 643$

2 (cont)

any factor of 90

either is divisible by 2 or not

& " " by 5 or not

& " " by 3 or 9 or not divisibly by 3

2	3	5	
2 ×	3 ×	5	30
		1	6
2 ×	9 ×	5	90
		1	18
2 ×	1 ×	5	10
		1	2
1	3	5	15
		1	3
1	9	5	45
		1	9
1	1	5	5
		1	1

12 factors

or $90 = 2^1 \times 3^2 \times 5^1$

$\Rightarrow (1+1)\times(2+1)\times(1+1) = 2\times3\times2 = 12$ factors

64 has 7 divisor

$64 = 2^6 \Rightarrow (1+6) = 7$ factors

123456 has 28 divisors

$123456 = 2^6 \cdot 3^1 \cdot 643^1 \Rightarrow (1+6)\times(1+1)\times(1+1) = 7\times2\times2 = 28$ factors

Algorithim for finding the factorization of a number n

need to know all primes up to $\sqrt{n}$ call these $p_1 = 2 \cdots p_m$

to factor n try to divide by p_1, if you can then $n = p_1 \times \frac{n}{p_1}$

& now factor $\frac{n}{p_1}$ by starting w/ p_1 & trial divisions

if not divisible by p_1 try p_2 & so on. if n isn't divisible by an n less then or equal to $\sqrt{n}$, then n is prime ex: $100 = n$ $\quad 2 \Rightarrow$

$$100 = 2 \times \frac{100}{2} = 2 \times 50 \quad | \; 2$$
$$50 = 2 \times \frac{50}{2} = 2 \times 25 \quad | \; 2$$

$\cancel{2}, \cancel{3}, 5 \Rightarrow$

$$25 = 5 \times \frac{25}{5} = 5 \times 5 \quad | \; 5$$
$$5 = 5 = 5 \times 1 \quad | \; 5$$

$$100 = 2^2 \cdot 5^2$$

ex $123456 = n$ $\quad 2 \Rightarrow$

$$123456 = 2 \times \frac{123456}{2} = 2 \times 61728 \quad | \; 2$$
$$61728 = 2 \times \frac{61728}{2} = 2 \times 30864 \quad | \; 2$$
$$30864 = 2 \times \frac{30864}{2} = 2 \times 15432 \quad | \; 2$$
$$15432 = 2 \times \frac{15432}{2} = 2 \times 7716 \quad | \; 2$$
$$7716 = 2 \times \frac{7716}{2} = 2 \times 3858 \quad | \; 2$$
$$3858 = 2 \times \frac{3858}{2} = 2 \times 1929 \quad | \; 2$$
$$1929 = 3 \times \frac{1929}{3} = 3 \times 643 \quad | \; 3$$
$$643 = \quad = 643 \times 1 \quad | \; 643$$

$\sqrt{643} = 25.357$

$2, 3, 5, 7, 11, 13, 17, 19, 23$

what if unique factorization was not true? could this happen

ex: let our set of numbers be the numbers of the form $3n+1$

$$A = \{1, 4, 7, 10, 13, \overset{4\times4}{16}, 19, 22, 25, \dots\}$$

in this set of #s 10 is prime, b/c its not a product of smaller #s in this set

in this world we dont have unique factorization

$$220 = 2 \times 110$$
$$= 2 \times 11 \times 10 = 2^2 \times 5 \times 11 \text{ (real world) } \not\in \text{ integers}$$
$$= 4 \times \underset{\Downarrow \atop [3\times18+1]}{55} = \quad \text{OR} \quad 22 \times 10$$