

Solving discrete problems — Math 2020, Spring 2005

Quiz, Thursday 27 January

Let a, b, c be positive integers.

Let P be the statement “ c divides a ”,

let Q be the statement “ c divides b ”,

let R the statement “ $a = cr$ for some positive integer r ”

let S the statement “ $b = cs$ for some positive integer s ”

let T be the statement “ $(a + b) = c(r + s)$ for some positive integers r and s ”

let U be the statement “ $(a - b) = c(r - s)$ for some positive integers r and s ”

let V be the statement “ $a + b$ is divisible by c ”

let W be the statement “ $a - b$ is divisible by c ”

Now write each of the following in symbolic notation, using the above symbols for each statement, and the connectives $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$.

If c divides a then $a = cr$ for some positive integer r ,
 $\underbrace{\hspace{10em}}_P \rightarrow \underbrace{\hspace{10em}}_R$

$$P \rightarrow R$$

If c divides b then $b = cs$ for some positive integer s ,
 $\underbrace{\hspace{10em}}_Q \rightarrow \underbrace{\hspace{10em}}_S$

$$Q \rightarrow S$$

If $a = cr$ and $b = cs$ for some positive integers r, s , then $a + b = c(r + s)$
 $\underbrace{\hspace{2em}}_R \wedge \underbrace{\hspace{2em}}_S \rightarrow \underbrace{\hspace{10em}}_T$

$$(R \wedge S) \rightarrow T$$

If $a = cr$ and $b = cs$ for some positive integers r, s , then $a - b = c(r - s)$
 $\underbrace{\hspace{2em}}_R \wedge \underbrace{\hspace{2em}}_S \rightarrow \underbrace{\hspace{10em}}_U$

$$(R \wedge S) \rightarrow U$$

$$T \rightarrow V$$

If $a + b = c(r + s)$ for some positive integers r, s , then $a + b$ is divisible by c
 $\underbrace{\hspace{10em}}_T \rightarrow \underbrace{\hspace{10em}}_V$

$$U \rightarrow W$$

If $a - b = c(r - s)$ for some positive integers r, s , then $a - b$ is divisible by c
 $\underbrace{\hspace{2em}}_U \rightarrow \underbrace{\hspace{10em}}_W$

$$(P \wedge (\neg Q)) \rightarrow (\neg V)$$

If a is divisible by c , and b is not divisible by c , then $a + b$ is not divisible by c .

If $a + b$ is not divisible by c then either a and b are both divisible by c , or they are both not divisible by c .
 $\underbrace{\hspace{10em}}_{\neg V} \rightarrow \underbrace{\hspace{10em}}_{(P \wedge Q) \vee (\neg P \wedge \neg Q)}$

$$(\neg V) \rightarrow ((P \wedge Q) \vee (\neg P \wedge \neg Q))$$

Note, all these statements are actually true, apart from the last.