Solving discrete problems — Math 2020, Spring 2005

Quiz, Thursday February 3

Theorem	
Assume a, b and p are integers	
If p is prime and	
If p divides ab	
then p divides a or p divides b.	
Proof	
If a is divisble by p, then the conclusion is true, so there's nothing more to do.	
Now we'll consider the case that p does not divide a.	
Since p is prime and a is not divisible by p, we know that $\gcd(a, p) = 1$.	
This implies that there are some numbers u, v with $au + pv = 1$.	
So $b(au + pv) = b$, and so $abu + bpv = b$, or rewriting, $b = abu + pbv$.	
Since ab is divisible by p, we can write $ab = pm$ for some m.	
From the previous two lines, $b = pmu + pbv = p(mu + bv)$.	
This implies that b is divisible by p.	
So, if a is not divisble by p, we've shown that b is.	
So either a or b is divisble by p. **QED**	

Questions:
1. In the statement of the theorem, write "H" in boxes at ends of lines containing a hypothesis.
2. In the statement of the theorem, write "C" in the box at the end of the line containing the conclusion.
3. In the proof, write "H" in the boxes on a line where a hypothesis is used.
4. In the proof, write "R" in lines in the where a previous result is used.
5. Find an example where all the hypothesis hold, and verify numerically that the conclusion is also true in this case.

6. Find an example where all hypothesis except one are true, and for which the conclusion is not true.