

Solution to 3rd

Quiz:

Compute the following modulo 17, giving your answer as a number between 0 and 16 (inclusive)

$$8 + 19 \pmod{17}$$

$$8 - 19 \pmod{17}$$

$$8 \times 19 \pmod{17}$$

$$8 / 19 \pmod{17}$$

Answer: 1) $(8 + 19) = 27 \equiv 27 - 17 \pmod{17}$
 $\equiv 10 \pmod{17}$

or $(8 + 19) \pmod{17}$
 $\equiv (8 \pmod{17}) + (19 \pmod{17}) \pmod{17}$
 $\equiv 8 + 2 \pmod{17}$
 $\equiv 10 \pmod{17}$

2) $(8 - 19) = (-11) \equiv -11 + 17 = 6 \pmod{17}$

or $8 - 19 \equiv x \pmod{17}$

means $8 - 19 - x$ is divisible by 17

which is same as saying $-8 + 19 + x$ is divisible by 17

so $11 + x$ is divisible by 17

x must be a positive number, or 0. the only value with $0 \leq x \leq 16$ is $x = 6$, since $11 + 6$ is divisible by 17

or $(8 - 19) \pmod{17} \equiv (8 \pmod{17}) - (19 \pmod{17}) \pmod{17}$
 $\equiv 8 - 2 \pmod{17}$
 $\equiv 6 \pmod{17}$

$$3) \quad 8 \times 19 = 152 = 17 \times 8 + 16 \equiv 16 \pmod{17}$$

$$\begin{aligned} \stackrel{or}{=} 8 \times 19 \pmod{17} &\equiv (8 \pmod{17}) \times (19 \pmod{17}) \\ &\equiv (8 \pmod{17}) \times (2 \pmod{17}) \\ &\equiv 16 \pmod{17}. \end{aligned}$$

(note, $16 \equiv -1 \pmod{17}$, but question asks for a positive number)

$$4) \quad 8/19 \equiv x \pmod{17}$$

means $8 \equiv 19 \times x \pmod{17}$, so $19x - 8$ is divisible by 17

can try values of x on calculator

$$19 \times 1 - 8 = 11$$

$$19 \times 2 - 8 = 38 - 8 = 30 = 17 + 13$$

$$19 \times 3 - 8 = 57 - 8 = 49 = 17 \times 2 + 11$$

$$19 \times 4 - 8 = 76 - 8 = 68 = 17 \times 4$$

since $19 \times 4 - 8$ is divisible by 17, $8/19 \equiv 4 \pmod{17}$

$$\stackrel{or}{=} (8/19) \pmod{17} \equiv \frac{8 \pmod{17}}{19 \pmod{17}} \equiv \frac{8}{2} \pmod{17} \equiv 4 \pmod{17}$$

$$\stackrel{or}{=} 8/19 \equiv x \pmod{17}$$

means $8 \equiv x \times 19 \pmod{17}$

$$\text{mean } 8 \pmod{17} \equiv (x \pmod{17}) \times (19 \pmod{17})$$

$$8 \pmod{17} \equiv (x \pmod{17}) \times (2 \pmod{17})$$

$$8 \equiv x \times 2 \pmod{17}$$

this has a solution $x = 4$.

Summary solutions are

10 mod 17, 6 mod 17, 16 mod 17, 4 mod 17