

Sept 12  
Monday

Due on Wednesday is Isidraff for groups  
FRIDAY redo inductions homework

BASE CASE

$$2. f_{n-1} \times f_{n+1} = f_n^2 + (-1)^n$$

$$f_{2-1} \times f_{2+1} = f_2^2 + (-1)^2$$

$$1 \times 2 = 1^2 + (-1)^2$$

$$2 = 2$$

Prove that  $P(n) \Rightarrow P(n+1)$

$$P(n) \text{ is } f_{n-1} \times f_{n+1} = f_n^2 + (-1)^n$$

GROUP  
WORK

### Induction Proofs for sums

Recall:

$$\textcircled{1} 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$\textcircled{2} 1 + x + x^2 + x^3 + \dots + x^{n-1} = \frac{x^n - 1}{x - 1}$$

$$\textcircled{3} \frac{1}{1 \cdot 2} +$$

typical form of such questions says

Prove that for some function  $f(i)$

we have  $f(1) + f(2) + \dots + f(n) = g(n)$

for some function  $g(n)$

$$1) f(i) = i \quad g(n) = \frac{n(n+1)}{2}$$

$$f(i) = x^{i-1}$$

$$g(n) = \frac{x^n - 1}{x - 1}$$

$$f(i) = \frac{1}{i(i+1)}$$

$$g(n) = \frac{n}{n+1}$$

Proof by induction

$P(n)$  is the statement  $\sum_{i=1}^n f(i) = g(n)$   
and want to prove this for all  $n$

$$f(1) + f(2) + f(3) = g(3) \quad | \quad n=3$$

$$f(1) + f(2) = g(2) \quad | \quad n=2 \rightarrow$$

$$f(1) = g(1) \quad | \quad n=1 \rightarrow$$

check that  $f(i) = g(i)$

by plugging in  $n=1$  for  
both sides

How do you get from one case to the  
next case?

$$f(1) + \dots + f(k) = g(k) \quad | \quad n=k$$

if we assume  $P(k)$  i.e.  $\sum_{i=1}^k f(i) = g(k)$   
we show  $P(k+1)$  is as follows:

assume  $f(1) + \dots + f(k) = g(k)$   $P(k)$  statement

Add another term

we want to show that  $f(1) + \dots + f(k) + f(k+1) = g(k) + f(k+1)$

prove this  
for any particular

example of  
 $P(k)$  then

$$g(k) + f(k+1) = g(k+1)$$

you proved by induction