

# Examples of relations

## Exercises.

**Q1.** The following table gives a list of 14 different relations.

Which of the relations are equivalence relations? Mark an  $X$  in the column under “is equivalence relation?” for those which are equivalence relations.

**Q2.** For those which are equivalence relations, give an example of an equivalence class (i.e., choose an element  $x$  of the set, and write a list of all the elements equivalent to  $x$ , or a formula for all the elements if this is an infinite set).

**Q3.** For those which are equivalence relations, in the last column of the table, write down the number of equivalence classes (this could be infinite.)

Each relation in this table is a relation on a given set, listed under “set”.

	set	relation	is equivalence relation?	number of equiv classes
1.	$\mathbf{R}$	$aRb \iff a - b$ is negative		
2.	$\mathbf{Z}$	$aRb \iff a - b$ is even		
3.	$\mathbf{R}$	$aRb \iff \sin(a) = \sin(b)$		
4.	$\mathbf{Z}$	$aRb \iff \sin(a) = \sin(b)$		
5.	$\mathbf{R}$	$aRb \iff \sin(\pi a) = \sin(\pi b)$		
6.	$\mathbf{Z}$	$aRb \iff \sin(\pi a) = \sin(\pi b)$		
7.	$\mathbf{R}$	$aRb \iff \lfloor a \rfloor = \lfloor b \rfloor$		
8.	$\mathbf{R}$	$aRb \iff  a  = b$		
9.	$\mathbf{R}$	$aRb \iff  a  =  b $		
10.	$\mathbf{R}$	$aRb \iff  a b = a b $		
11.	$\mathbf{Z}$	$aRb \iff  a  -  b $ is even		
12.	$\mathbf{Z}$	$aRb \iff a b$		
13.	$\mathbf{Z} \setminus \{0\}$	$aRb \iff ab$ is a square in $\mathbf{Z}$		
14.	$\mathbf{R} \setminus \{0\}$	$aRb \iff ab$ is a square in $\mathbf{R}$		

Note, in examples 3, 4, 5, 6,  $a$  and  $b$  are in radians.

Note, in example 7., the  $\lfloor x \rfloor$  is the largest integer smaller than  $x$ , e.g.,  $\lfloor 2.3 \rfloor = 2$ , or  $\lfloor 7.9 \rfloor = 7$ ,  $\lfloor -2.3 \rfloor = -3$ .

This is also called rounding down to the integer part of  $x$ .