$37^{(4)}$ or $38^{(2)}$ = A
$34^{(3)}$ or $35^{(2)}$ = B
$27^{(1)}$ = C
$17^{(1)}$ = D

each questn graded out of 10.
grade entered will be
$((10 + \text{Score}) \times 2)$ %

Test 1 for Math4181. October 5, 2007. All questions equal weight.

Note, solutions to (1) & to (3) are not unique, so your answers could be correct even if not equal to solutions below.

(1) Find $(72, 32)$, and find integers s, t such that
$$72s + 32t = (72, 32)$$
(2) Prove that for $a, n \in \mathbf{Z}$, $n > 1$,
$$(a, n) = 1 \iff \bar{a} \text{ is not a zero divisor in } \mathbf{Z}_n$$
(3) Find all the units in $\mathbf{Z}_{14}$, and give their inverses, in the form $\bar{a}$ where $a \in \mathbf{Z}$ and $0 \le a < 14$.
(4) Is U_{14}, the group of units in $\mathbf{Z}_{14}$, cyclic? If it is, give a generator, and if not, exaplain why not.

1) let $a = 32$, $b = 72$ write $b = aq + r$
$72 = 32 \times 2 + 8$ so $(32, 72) = (32, 8)$.
since $8 | 32$, $(32, 8) = 8$.
So $(72, 32) = 8$. $72 - 32 \times 2 = 8$
so take $s = 1$, $t = -2$.

2) $\Rightarrow$ suppose $(a, n) = 1$. Suppose $\bar{a}\bar{b} = \bar{0}$ for some $\bar{b} \in \mathbb{Z}_n$
so $n | ab$ but $(a, n) = 1$
so there are $s, t \in \mathbb{Z}$ such that $as + tn = 1$
and for some $q \in \mathbb{Z}$ $ab = nq$ so $bas + btn = b$
$= nqs + btn$
$= n(qs + bt) = b \Rightarrow b | n$
so if $\bar{a}\bar{b} = \bar{0}$, then $\bar{b} = \bar{0}$ in $\mathbb{Z}_n$, so $\bar{a}$ is not a zero divisor

$\Leftarrow$ suppose $(a, n) = d \neq 1$
then $\cancel{\frac{a}{a}} \circ \bar{a} \circ \frac{\bar{n}}{d} = \frac{\bar{a}}{d} \circ \bar{n} = \bar{0}$ so $\bar{a}$ is a zero divisor, since $\frac{n}{d} \not\equiv 0 \mod n$, since $d > 1$.
So if $\bar{a}$ is not a zero divisor $(a, n) \neq 1$.
QED.

3) Units in $\mathbb{Z}_{14}$: $\bar{1}, \bar{3}, \bar{5}, \bar{9}, \overline{11}, \overline{13}$. $\bar{1}^{-1} = \bar{1}$, ~~$\bar{3}^{-1} = \bar{9}, \bar{9}^{-1} = \bar{3}$ since $3 \times 9 = 13 \times 2 + 1$~~
$\overline{13}^{-1} = \overline{13}$ since $13 \times 13 = 169 = 14 \times 12 + 1$
$\bar{3} . \bar{5} = \overline{15} = \bar{1}$, $\bar{9} \times \overline{11} = \overline{99} = \overline{7 \times 14 + 1} = \bar{1}$

so

$$\boxed{\begin{array}{ll} \bar{1}^{-1} = 1 & \bar{9}^{-1} = \overline{11} \\ \bar{3}^{-1} = \bar{5} & \overline{11}^{-1} = \bar{9} \\ \bar{5}^{-1} = 3 & \overline{13}^{-1} = \overline{13} \end{array}}$$

4) in U_{14}, $\bar{3}^2 = \bar{9}$, $\bar{3}^3 = \overline{27} = \overline{13}$, $\bar{3}^4 = \bar{3} . \overline{13} = \overline{39} = \overline{11}$, $\bar{3}^5 = \bar{3} . \overline{11} = \overline{33} = \bar{5}$, $\bar{3}^6 = \bar{3} . \bar{5} = \bar{1}$, So U_{14} is cyclic generated by $\bar{3}$