PERSONAL STATEMENT OF RESEARCH INTERESTS

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1. Current research

It is well known that any knot $K \subset S^3$ may be unknotted by a sequence of crossing changes. A crossing change may be obtained by performing ± 1-framed surgery on S^3 along an unknot, in the complement of K, which loops once around both strands of the crossing. That is, any knot may be obtained from an unknot in the 3-sphere by ± 1-framed surgery along null-homotopic circles in the complement of the unknot. This idea is called the *surgery description* of a knot. My current research involves the analog of surgery description for *p-colored knots* for p an odd prime.

A p-colored knot is a pair (K, ρ) consisting of a knot $K \subset S^3$ and a surjective map ρ from the knot group onto a dihedral group. D. Moskovich conjectures that for any odd prime p there are exactly p equivalence classes of p-colored knots up to surgery along unknots in the kernel of the coloring (see his paper [Mos1]).

The pair (M, φ) consisting of a closed 3-manifold M and a map φ from itself to an $Eilenberg-Maclane$ space $K(G, 1)$ represents an element of the *bordism group* $\Omega_3(G)$. In the case that G is the quotient of the fundamental group of M by some normal subgroup, T. Cochran, A. Gerges, and K. Orr show that $\varphi_*([M]) \in H_3(K(G,1); \mathbb{Z})$, where $[M]$ denotes the fundamental class, is an invariant of the *surgery equivalence* class of M [CGO]. The *colored untying invariant* $cu(K, \rho)$ is a computable invariant of the *surgery equivalence class* of a p-colored knot. This may be defined in the same way as the Cochran-Gerges-Orr invariants when we take M to be 0-framed surgery of S^3 along K and the normal subgroup of $\pi_1(M)$ to be the kernel of the coloring ρ. In my paper, *Surgery description of colored knots*, I show that there are at most $2p$ equivalence classes of p-colored knots up to surgery along unknots in the kernel of ρ. This is an improvement upon the previous results by Moskovich for $p = 3$, and 5, with no upper bound given in general.

Theorem 1.1. *There are at most $2p$ surgery equivalence classes of p-colored knots. Moreover, if K_p denotes the left-handed $(p, 2)$-torus knot and ρ is any non-trivial coloring for K_p then*

$$(K_p, \rho), (K_p, \rho)\#(K_p, \rho), \ldots, \#_{i=1}^{p}(K_p, \rho)$$

are p distinct surgery classes.

I do not attempt a direct proof of Theorem for $p > 5$ as Moskovich does for the first two cases. Instead, I show that an analog to the *Lickorish-Wallace Theorem* and some basic bordism theory suffices to show that there are no more than $2p$ classes.

2. Immediate future research

The outright proof of Moskovich's conjecture is the immediate goal. The conjecture, if true, would establish an algorithm for finding a surgery diagram for the *dihedral irregular branched covers* of S^3. The procedure is outlined in Moskovich's original "coloured untying" paper and explained in greater detail in his paper [Mos2]. The next case to consider would be to establish a similar result arising from representations of knot groups onto the alternating group A_4.

My bordism approach for the dihedral cases seems best when trying to generalize the result for other target groups. My conjecture is that there are two surgery equivalence classes of A_4-*colored knots*. Unfortunately my only evidence of this so far is that any A_4-coloring of a knot has a natural dual arising from the fact that there are two conjugacy classes of 3-cycles in the group. The immediate goal is to study the bordism group of A_4 relative a prescribed $\mathbb{Z}_3$ subgroup which would be a direct analog to the dihedral case. The hope is that a *linking form* on the *triple-branched cover* of S^3 along the A_4-colored knot may indeed be a surgery equivalence invariant.

3. Farther into the future research

There are several problems left to tackle in the dihedral cases. For instance, what are the surgery equivalence classes for a n-colored knot where n is not a prime? Also, what, if any, would be a sufficient list of "surgery equivalence moves" on diagrams as an analog to the Reidemeister moves? As another long-term goal, I would like to extend the notion of knot concordance to colored knots. I am also interested in *quandles* and *quandle colorings*. Some of the research done by Sam Nelson and his co-authors on classification of different classes of quandles and biquandles seems like a possible avenue for research in the future. I believe that these long-term directions have potential to create problems for both undergraduate and graduate student research.

References

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[LiWa] R.A. Litherland, S.D. Wallace, *Surgery description of colored knots*. Preprint arXiv:0709.1507.

[Mos1] D. Moskovich, *Surgery untying of coloured knots*. Alg. Geom. Topol. **6** (2006), 673-697.

[Mos2] D. Moskovich, *Surgery Description of Orientation-Preserving D_{2p}-Actions on Compact Orientable 3-Manifolds*. arXiv:math/0508141v2.

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