Answers to Test #3

Problem 1: Coefficients are $a = -2$, $b = 8$ and $c = -6$. The parabola **opens down** because $a = -2 < 0$. We find vetrex as point (h, k), where $h = -\frac{b}{2a}$ and $k = p(h)$. We have:

$$h = -\frac{8}{-4} = 2,$$

$$k = p(h) = p(2) = -2 \cdot 2^2 + 8 \cdot 2 - 6 = 2.$$

Axis of symmetry is line $x = h$, that is $x = 2$. y-intercept is $p(0) = -6$ and x-intercepts are solutions of $p(x) = 0$:

$$p(x) = -2(x-2)^2 + 2,$$
$$-2(x-2)^2 + 2 = 0,$$
$$(x-2)^2 = 1,$$
$$x_{1,2} = 2 \pm 1.$$
$$x_1 = 3 \quad x_2 = 1,$$

Thus x-intercepts are the points $(1;0)$ and $(2;0)$. Domain is all numbers. Range is interval $(-\infty, 2]$. Function increases on $(\infty, 2]$ and decreases on $[2, \infty)$.

Problem 2:

(a) $R(x) = xp(x) = -\frac{1}{2}x^2 + 50x$

(b) $R(10) = -\frac{1}{2}100 + 500 = 450$

(c) Quantity $x_{\max} = -\frac{b}{2a}$, where $a = -0.5$ and $b = 50$ are coefficients of the quadratic function $R(x)$. So,

$$x_{\max} = -\frac{50}{-\frac{2}{2}} = 50.$$

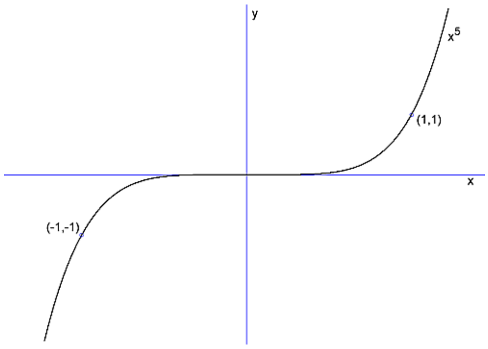
Since

$$R(x_{\max}) = R(50) = -\frac{1}{2}2500 + 2500 = 1250,$$

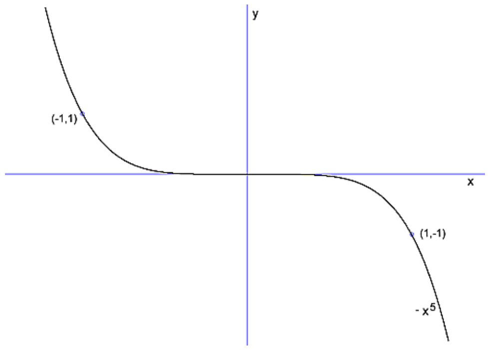
we conclude that the maximum revenue of 1250 dollars is attained when 50 items are sold.

(d) Since $p(x_{\max}) = p(50) = -\frac{1}{2}50 + 50 = 25$, the price of 25 dollars per item should be charged to reach the maximum revenue.

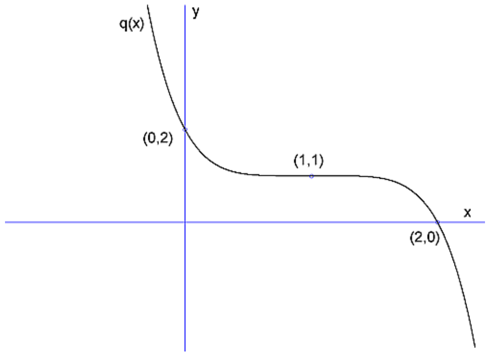
Problem 3: We first plot power function x^5 :



Next step, flip x^5 around x-axis to obtain $-x^5$



To obtain $q(x)$ move $-x^5$ to the right by one and then to the upwards by one.



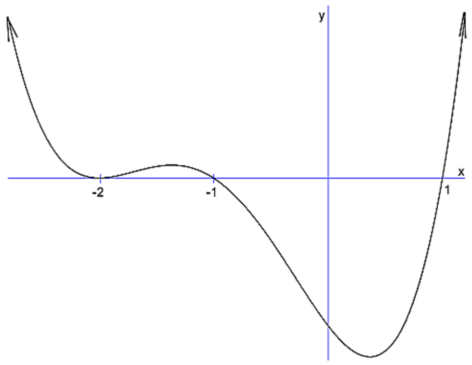
Problem 4:

(a) y-intercept is $f(0) = -1 \cdot 1 \cdot 4 = -4$. x-intercepts are the following points $(1;0)$, $(-1;0)$ and $(-2,0)$, which is directly read from $f(x)$ since it is a factored polynomial function.

(b) Multiplicity of "zeros" 1 and -1 is odd, so the graph crosses x axis at those points. Multiplicity of "zero" -2 is even, so the graph touches x axis at that point.

(c) The highest power of $f(x)$ is 4, so power function x^4 resembles f for large values of $|x|$.

(d) Function $f(x)$ is positive on intervals $(-\infty, -2)$, $(-2, -1)$ or $(1, \infty)$. Function $f(x)$ is negative on interval $(-1, 1)$.

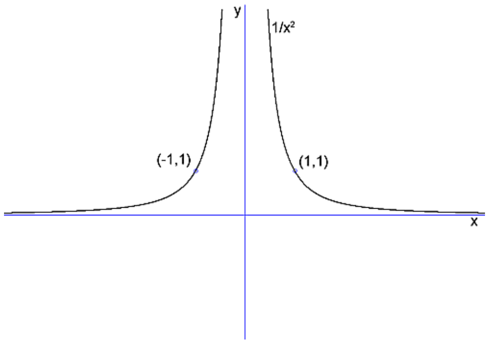


Problem 5:

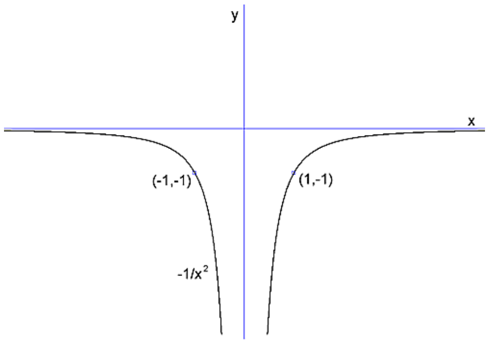
$$r(x) = -\frac{1}{x^2 + 4x + 4} = -\frac{1}{(x+2)^2}.$$

Flip $1/x^2$ around x-axis, and then move it to the left by 2. Function $1/x^2$ has horizontal asymptote $y = 0$ and it doesn't change during the transformation, so $r(x)$ also has it as an asymptote. However, vertical asymptote $x = 0$ of $1/x^2$ moves to the left by 2, so vertical asymptote for $r(x)$ is vertical line $x = -2$. There are no x-intercepts and there is one y-intercept, $r(x) = -1/4$. Function $r(x)$ is never positive.

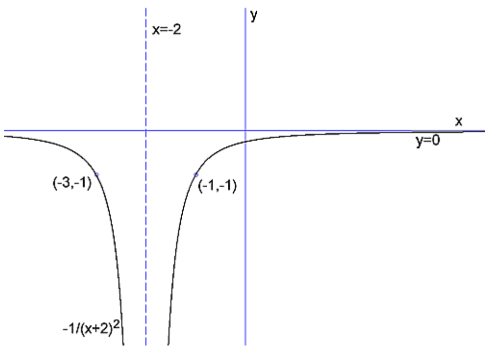
Steps in plotting $r(x)$ are as follow. We first plot $1/x^2$, which we know from the library of elementary functions...



then we flip it around the x-axis...



and we obtain $r(x)$ by shifting the previous graph by two units to the left.



Problem 6: Solve the following inequality: Let $F(x) = (x+3)(x-2)(x-3)$. We need to find for what x $F(x)$ is positive.

zeros	-3	2		3
intervlas	$(-\infty, -3)$	$(-3, 2)$	$(2, 3)$	$(3, +\infty)$
points	-4	0	2.5	4
$F(x)$	-42	18	$-11/8$	14
behavior	negative	positive	negative	positive

Inequality holds on interval $(-3, 2)$ or interval $(3, +\infty)$.

Problem 7: Yes, $x - 1$ is a factor of $h(x)$ because $h(1) = 0$. Since $h(x)$ has three variations of sign, there are either one of three positive zeros. Since $h(-x) = 2x^4 + 2x^3 + 3x^2 + 2x - 1$ has only one change of sign, there is only one negative zero. Confirm that $x - 1$ is factor of $h(x) = 2x^4 - 2x^3 + 3x^2 - 2x - 1$. Since number 1 factors as $p := \pm 1$ and number 2 factors as $q := \pm 1, \pm 2$, all possible rational zeros are

$$\frac{p}{q} = \pm 1, \pm\frac{1}{2}.$$

Problem 8: Substitute $s = x^2$. We get

$$s^2 + 2s - 8 = 0.$$

Using the quadratic formula, we solve this equation for s:

$$s_{1,2} = \frac{-2 \pm \sqrt{4+32}}{2} = \frac{-2 \pm 6}{2}.$$
$$s_1 = 2, \quad s_2 = -4.$$

Now we find x by using established relation $s = x^2$.

$$x^2 = 2 \quad \text{or} \quad x^2 = -4.$$

Since $x^2 = -4$ doesn't have any solutions, and $x^2 = 2$ implies $x_{1,2} = \pm\sqrt{2}$, we determine that the original equation has two real solutons $\sqrt{2}$ and $-\sqrt{2}$. In addition, the equation factors as

$$(x - \sqrt{2})(x + \sqrt{2})(x^2 + 4) = 0.$$

Problem 9:

$$(g \circ h)(x) = g(h(x)) = g(-x) = -2x + 1$$
$$(h \circ f)(x) = h(f(x)) = h(x^2 + \frac{1}{2}) = -x^2 - \frac{1}{2}$$