Research Statement

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I have both theoretical and applied research interests. The theoretical part of my research is based around my PhD thesis and my applied interests pertain to several health industry projects including cancer research. In this letter I will briefly describe the main points of both areas of my research interest.

1. Impulsive Systems. In my research of optimal control theory with impulsive systems I use modern techniques of nonsmooth analysis [**6, 7**]. Impulsive systems arise in a variety of applications where states can move at different time scales. The "slow" movement can be thought of as the usual time progression infinitesimally incremented by dt, and the "fast" movement occurs over a small interval that resembles the effect of a point-mass measure.

Engineers often refer to these types of systems as hybrid systems [**10**], but in their literature "slow movements" and "fast movements" are observed as two separate differential equations. Such an approach was also presented in [**11**]. My research adopts the mathematical formalism proposed by various authors such as Miller, Rubinovich, Vinter, Pereira and Silva [**12, 17, 18, 19**], where two dynamics are expressed in one single equation. A dynamic system is the sum of a slow-time velocity belonging to a set $F(x)$ and a fast-time contribution coming from another set $G(x)d\mu$, where μ is a vector valued measure. Additionally, dynamics are represented with a differential inclusion, rather than a differential equation.

In particular, the following differential form describes the impulsive systems considered in my research

$$\begin{cases} dx & \in & F\big(x(t)\big)\,dt + G\big(x(t)\big)\,d\mu(dt) \\ x(0-) & = & x_0. \end{cases}$$

Here, measure μ belongs to the set of vector-valued Borel measures defined on the interval $[0,T] \subset I\!R$ with values in a closed convex cone $K \subseteq I\!R^m$. Multifunctions $F : I\!R^n \rightrightarrows I\!R^n$ and $G : I\!R^n \rightrightarrows \mathcal{M}_{n\times m}$ (where $\mathcal{M}_{n\times m}$ denotes the $n \times m$ dimensional matrices with real entries) are with closed graph and convex values, and satisfy the linear growth condition.

My research in impulsive systems first intrigued me after being exposed to the work of the Italian mathematicians Bressan and Rampazzo [**1, 2, 3, 13, 15**], who emphasized that issues arise to the nature of a "solution" in nonlinear systems when the fast dynamic velocities are affected by multiplicative state dependence (i.e. G depends nontrivially on x), and that the solution concept will only be well defined if a realization of the measure is also prescribed through a graph completion. Following the idea of graph completion, modern research in impulsive systems is based on the idea of time reparametrization. Solution concepts in contemporary research and the various results in this area have been expressed in reparameterized time, rather than original time. The utilization of original time is preferable, especially when dealing with various optimality issues such as minimum time function and my contribution has been to formulate a theory in which all expressions are stated in original time.

The essence of this theory is a new solution concept to an impulsive system that has been proven to agree with the appropriate modification of the Bressan-Rampazzo solution. This novel concept requires a direct correlation respectively of the absolutely continuous, continuous singular, and atomic parts of the bounded variation solution and the given measure. I refer to it as a *direct solution.* However, the main result is a sampling method that is analogous to the classical Euler one-step method for the non-impulsive systems [**6, 7**].

There are essentially two ways to sample impulsive systems, one with measure μ and its completion specified and the other which actually produces measure μ and its completion. I consider both types of sampling methods in my research.

My other research results in the theory of impulsive systems are related to Hamilton-Jacobi theory and include invariance and minimum time theorems. Such theory has already been well developed for non-impulsive systems (see [**6**]), but there remains needed research for the impulsive case. I intend to generalize various forms of optimality conditions and investigate what role they play in impulsive systems.

Murray [**14**] extended integral functionals of generalized variational problems from absolutely continuous functions to ones of bounded variation through infinite penalizations, and made similar conclusions about reparemeterization and completions like Bressan and Rampazzo. My plan is to further investigate the relation between these two approaches as they are clearly related. Furthermore, I propose to analyze particular occurrences in biology that can be described with impulsive systems. I also intend to explore relations between impusive systems and systems with singluar pertubations. Another possible direction is to investigate the properties of impulsive systems within stochastic framework.

My research is constantly evolving; I am regularly identifying new areas of interest within the theory of impulsive systems and I am considering new approaches to improve upon my theory. To this end, I fully intend to continue my collaboration with Professors Peter Wolenski, my doctoral adviser at Louisiana State University, and Fernando Lobo Pereira, to whom I was a research assistant for one semester at the University of Portugal.

The results I have achieved to date in my work dedicated to the theory of impulsive systems are described in detail in my PhD thesis and I expect to complete my doctoral degree in May 2005. Various elements of my doctoral research appear in three separate papers, one of which has already been published in Proceedings of Controlo 2004 [**20**], other [**21**] has been submitted to SIAM Journal on Control and Optimization, and the third one [**22**] is in the final phase of preparation and it will soon be submitted as well.

2. Mathematical Modeling In Health. In addition to my work in the theoretical nonsmooth analysis, I dedicated much time to mathematical modeling in health industry. Department of Mathematics at LSU is requently contacted by the Kinesology Department at LSU and local health centers such as Lake PET Imaging Center and LSU Eye Center asking help in their research. Through my studies at LSU I have been heavily involved in many of those projects.

The most challenging project related to mathematical modeling that I have been involved with has been the processing of fused sets of images that were acquired using Positron Emission Tomography scans (PET) and Computed Tomography scans (CT). The fused PET/CT images in my research were obtained at Lake PET Imaging Center of patients with an established cancer diagnosis. My research in this area centers on creating and validating a computer algorithm that detects the anatomic edges of a tumor in fused PET/CT data.

Modern radiation therapy planning systems typically use only CT scans to direct beams. CT scans provide three-dimensional (3-D) images that are used as a road map for both targeting tumor and avoiding normal tissue. Recently PET scans which are also 3-D images have been shown to have higher accuracy in detecting tumors. The CT scans show anatomical parts of the body and the PET scans show their physiological activity. Both are quantitative tomographic techniques. However, neither of the two scans are enough to correctly outline the tumor with a region of interest (ROI). Fusion of CT and PET scans has been successfully used by physicians to prescribe a radiation treatment plan (RTP) [**5**]. A great deal of user subjectivity is introduced when the qualitative information of a PET scan is to be translated into a target volume, as PET scans lack sharp edges.

Image processing using the algorithm reveals more accurately the edges of the tumor allowing the radiation oncologists to minimize the negative side effects on healthy tissue surrounding the tumor and direct radiation optimally. A successful algorithm would have a wide application in RTP and would greatly enhance the value of medical data acquired by PET/CT scans. This project was initialized by Steven Bujenović, MD, Director of Lake PET Imaging Center, an outpatient clinic in Baton Rouge, Louisiana, specializing in PET/CT imaging.

The algorithm I proposed is a combination of edge detection methods and it was implemented in MATLAB programming language. Input images were taken by a CTI REVEAL HD PET/CT camera. We tested the algorithm on twelve PET/CT images and algorithm showed excellent execution speed, very good results for the tested neck tumors and useful, but slightly less accurate results for mediastinal tumors in the chest. Overall, the computer-generated parametric images were clinically helpful in targeting hypermetabolic activity within the tumors, but the size of the gross tumor value (GTV) was slightly underestimated in some cases. I intend to evaluate and improve my algorithm in upcoming phantom studies. I also wish to use these detailed results in image processing to improve current RTP optimization techniques. This will require a new mathematical model for linear accelerators delivering lethal dosage of radiation to the tumor while avoiding the healthy tissue.

Results of my cancer research are not published yet, but they were presented on a meeting of radiation oncologists at University of Belgrade in Serbia. Also, my talk on these results has been accepted and will be presented in March 2005, at the Annual Conference of the Academy of Molecular Imaging in Orlando, Florida [**4**].

My past research in math modeling for health industry also included image processing in which optical nerve head change was successfully detected using the proper orthogonal decomposition. I was a member of a team that consisted of one clinician, one computer scientist, and two mathematicians (one of which was myself). We created an algorithm that was clinically tested and now it is being used at the LSU Eye Center in New Orleans. Also, I worked with several kinesiologists on the signal processing of heart rate data for the Kinesiology Department at LSU. Heart rate data was analyzed with respect to the fractal dimension and frequency spectrum attained using the wavelet and Fourier decompositions. This spectrum data has been successfully used in several different projects at the Kinesiology Department [**8, 9, 16**].

I am versed in computer programming and I worked in programming lanuages such as MatLab, C++, Pascal, FORTRAN and used programming packages such as OPL Studio, Maple and Mathematica.

My goal is to pursue my research interests and to build on my ideas and experience with new topics.

References

[1] A. Bressan, *Impulsive control systems*, in "Nonsmooth Analysis and Geometric Methods in deterministic Optimal Control," B. Mordukhovich and H. Sussmann, eds., 1-22.

[2] A. Bressan and F. Rampazzo, *Impulsive systems with commutative vector fields*, J. of Optimization Theory and Applications, v. 71, (1991), 67-83.

[3] A. Bressan and F. Rampazzo, *Impulsive control systems without commutativity assumptions*, J. of Optimization Theory and Applications, v. 81, (1994),435-457.

[4] L.S. Bujenović and S. Žabić, *A Computer algorithm for gross tumor volume determination in PET/CT images for radiation treatment planning* (in preparation)

[5] L.S. Bujenović, *The Role of Positron Emission Tomography in Radiation Treatment Planning*, Seminars in Nuclear Medicine, v.6 2004, 293-299.

[6] F. H. Clarke, Yu. S. Ledyaev, R. J. Stern, P. R. Wolenski, *Nonsmooth Analysis and Control Theory*, Springer, New York (1998).

[7] F. H. Clarke, *Optimization and Nonsmooth Analysis*, Canadian Mathematical Society, Wiley-Interscience, Toronto, (1983).

[8] J. DeRouen, R. Wood, M. Welsch, P.Wolenski, S. Žabić, *Discrimination of Cardiac health and disease by assessment of heart rate variability: Wavelet vs. FFT*, Medicine and Science in Sports and Exercise 34(5): S-147 (2002).

[9] H.A. Kluess, R.H. Wood, D.J. Stone, S. Žabić, P.Wolenski, M.A. Welsch, *Vagal modulation and central hemodynamic response to common doses of orally administered aspirin during dynamic handgrip exercise and forearm occlusion*, Proceedings of the Annual Meeting of the Southeastern Chapter of American College of Sports Medicine (2002).

[10] A.S. Matveev and A.V. Savkin *Qualitative Theory of Hybrid Dynamical Systems.* Birkhäuser, Boston (2000).

[11] V. Lakshmikantan, D.D. Bainov, P.S. Simeonov, *Theory of Impulsive Differential Equations.* Singapore, World Scientific (1989).

[12] B.M. Miller and E.Ya. Rubinovich, *Impulsive control in continuous and discontinuous systems (Foundations of the hybrid systems theory)*, Kluwer Academic/Plenum Publishers, New York (2002)

[13] M. Motta and F. Rampazzo, *Dynamic programming for nonlinear systems driven by ordinary and impulsive control*, SIAM J. Control and Optimization, v. 34 (1996), 199-225.

[14] J.M. Murray, *Existence theorems for optimal control and calculus of variations problems where the states can jump*, SIAM J. Control, v. 24 (1986), 412-438.

[15] F. Rampazzo and C. Sartori, *The minimum time function with unbounded controls*, J. of Math. Systems, Estimation and Control, v. 8 (1998), 1-34.

[16] R. Reyes, M. Alomati, S. Žabić, L. Li, M. Welsch, R. Wood, *Post-exercise hypotension after resistance exercise and dmished vascular responsiveness in older women*, Medicine and Science in Sports and Exercise 36(5), (in press).

[17] G.N. Silva, and R.B. Vinter, *Measure driven differential inclusions.* J. of Mathematical Analysis and Applications, v. 202 (1996), 767–746.

[18] G.N. Silva and R.B. Vinter, *Necessary conditions for optimal impulsive control systems*, SIAM J. on Control and Optimization, v. 35 (1998), 1829-1846.

[19] R.B. Vinter and F.M.F.L. Pereira, *A maximum principle for optimal processes with discontinuous trajectories.* SIAM J. of Control and Optimization v. 26 (1988), 205–229.

[20] P.R. Wolenski, S. Žabić, *Direct solution concept for impulsive systems.* Proceedings of Controlo 2004, Faro (2004), vol.2.

[21] P.R. Wolenski, S. Žabić, *A solution concept and sampling method for impulsive systems.* SIAM J. of Control and Optimization (submitted).

[22] P.R. Wolenski, S. Žabić, *Invariance principle for impulsive systems.* (in preparation).