Answers to test #4

Problem 1: $f(x) = 1 - 3x$.

a) Equation related to $f(x)$ is $y = 1 - 3x$. We solve if for x and get $x = 1/3 - y/3$. For every fixed value of y, we get only one x, thus by the horizontal line test, $f(x)$ is one-to-one.
b) $f^{-1} = 1/3 - x/3$.
c) $f(f^{-1}(x)) = f(1/3 - x/3) = 1 - 3(1/3 - x/3) = 1 - 1 + x = x$.
$f^{-1}(f(x)) = f^{-1}(1 - 3x) = 1/3 - (1 - 3x)/3 = 1/3 - 1/3 + x = x$.

Problem 2:

b) Only one asymptote, $y = -1$.
c) y intercept: $g(0) = e^2 - 1$.
x intercept: $e^{x+2} - 1 = 0$, $x + 2 = 0$, $x = -2$.
d) Domain is all numbers.
e) Range is interval $[-1, +\infty]$.
f) $g(2) = e^4 - 1 = 53.598$.

Problem 3:

$$5^{1-3x} = \frac{1}{25},$$
$$5^{1-3x} = 5^{-2},$$
$$1 - 3x = -2,$$
$$-3x = -3,$$
$$x = 1.$$

Problem 4:

a) $x = \log_5 2$
b) $y = 3^{-2}$

Problem 5: $h(x) = \ln(x + 3)$

c) x intercept: $h(x) = 0$, $\ln(x+3) = 0$, $x+3 = 1$, $x = -2$.
y intercept: $h(0) = \ln 3$
d) Domain is all x such that $x + 3 > 0$. That is, domain is interval $(-3, \infty)$.
e) Range is all numbers.

Problem 6:

$$3\ln(x - 2) = 6,$$
$$\ln(x - 2)^3 = 6,$$
$$(x - 2)^3 = e^6,$$
$$x - 2 = e^2,$$
$$x = e^2 + 2.$$

Problem 7:

a) False
b) True
c) False
d) True
e) False

Problem 8:

$$2\log_3 2x - \log_3 36 = 0,$$
$$\log_3 4x^2 = \log_3 36,$$
$$4x^2 = 36,$$
$$x^2 = 4,$$
$$x = \pm 2,$$

But, since $\log_3 2x$ is not defined for $x = -2$, this result is extreneous, and we conclude, $x = 2$. is the solution of this equation.

Problem 9:

$$\ln(5x - 3) - \ln(x + 1) = 2,$$
$$\ln\left(\frac{5x - 3}{x + 1}\right) = 2,$$
$$\frac{5x - 3}{x + 1} = e^2,$$
$$5x - 3 = e^2(x + 1),$$
$$5x - e^2 x = e^2 + 3,$$
$$x(5 - e^2) = e^2 + 3,$$
$$x = \frac{e^2 + 3}{5 - e^2}.$$

Problem 10:

$$3^{2x} = 5^{(x+3)},$$
$$2x\ln 3 = (x + 3)\ln 5,$$
$$x(2\ln 3 - \ln 5) = 3\ln 5,$$
$$x = \frac{3\ln 5}{2\ln 3 - \ln 5},$$

Problem 11:

$$P = P_0\left(1 + \frac{r}{n}\right)^{tn},$$
$$P = 150\left(1 + \frac{0.0399}{12}\right)^{3\cdot 12},$$
$$P = 169.0402.$$

After three years we have 169 dollars and 4 cents.

Problem 12: We use information about the half-life to compute the rate of decay:

$$0.5A_0 = A_0 e^{5.27r}.$$
$$0.5 = e^{5.27r}.$$
$$\ln 0.5 = 5.27r.$$
$$r = \frac{\ln 0.5}{5.27}.$$

Now we find how much is left after 20 years:

$$A = 100e^{rt},$$
$$A = 100e^{20\frac{\ln 0.5}{5.27}},$$
$$A = 7.204.$$

There will be 7.204 grams of cobalt left in twenty years from now.