Answer sheet to Test #2

Problem 1:

$$\text{(a)}\quad 3y = 6x + 1 \quad | : 3$$
$$y = 2x + \frac{1}{3}.$$
$$m = 2.$$

(b) $y = -2$ is a horizontal line, so slope is $m = 0$.

Problem 2: Find point-slope equations of the line:

Point=slope forumla has a following form:

$$y - y_1 = m(x - x_1).$$

$$\text{(a)}\quad m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$x_1 = -1, \quad x_2 = 3,$$
$$y_1 = -3, \quad y_2 = 2.$$
$$m = \frac{2+3}{3+1} = \frac{5}{4},$$
$$y + 3 = \frac{5}{4}(x+1).$$

$$\text{(a)}\quad m = \frac{1}{2}, \quad C = (2,1).$$
$$y - 1 = \frac{1}{2}(x-2).$$

Problem 3: General slope-intercept formula: $y = mx + b$. Slope of the line $2x + y = 0$ is -2. So, the slope of the line that we are looking for is negative reciprocal of -2, $m = \frac{1}{2}$. Now we need to find b using the point $(2, 0)$. Plug $y = 0$ and $x = 2$ into $y = \frac{1}{2}x + b$. We get

$$0 = \frac{1}{2}x + b,$$
$$0 = 1 + b,$$
$$b = -1.$$

The equation of the line that we are looking for is

$$y = \frac{1}{2}x - 1.$$

Problem 4:

$$x^2 + 6x + (y-2)^2 = 7,$$
$$x^2 + 6x + 9 + (y-2)^2 = 7 + 9,$$
$$(x+3)^2 + (y-2)^2 = 16.$$

We read the center and radius directly from the formula, center is $C = (-3, 2)$, radius is $r = 4$.

Problem 5:

a)

$$x - y = 0$$
$$y = x$$
$$f(x) = x$$

This is a function.

b)

$$y^2 + x^3 = 1$$
$$y^2 = 1 - x^3,$$
$$y = \pm\sqrt{1 - x^3}.$$

When $x = 0$, then $y = 1$ and $y = -1$. Equation fails the vertical line test, so this is not a function.

Problem 6: This function is defined on all numbers except $x = 2$ and $x = -2$.

Problem 7: Consider $f(x) = 1 - 5x$.

a) $f(2) = 1 - 10 = -9$,

b)

$$f(x+h) = 1 - 5(x+h),$$
$$f(x+h) = 1 - 5x - 5h.$$
$$f(x+h) - f(x) = 1 - 5x - 5h - 1 + 5x = -5h.$$
$$\frac{f(x+h) - f(x)}{h} = \frac{-5h}{h} = -5.$$

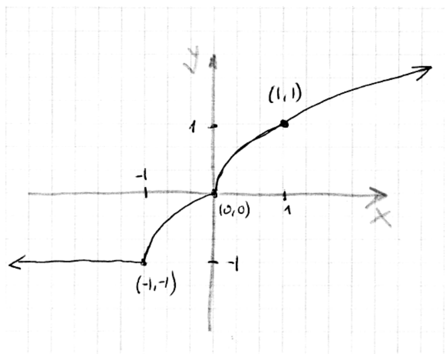
Problem 8: Consider the following graph:

a) $f(-3) = -3$ and $f(6) = 3$.
b) $f(x) = 0$ for $x = -8$, $x = -4$ and $x = 5$,
c) $f(x) > 0$ for all x belonging to intervals $(-8, -4)$ and $(5, 8)$.
d) Domain is the interval $[-8, 8]$.
e) Range is the interval $[-6, 3]$.
f) On interval $[6, 8]$ the function is constant.

Problem 9: Consider the following graph:

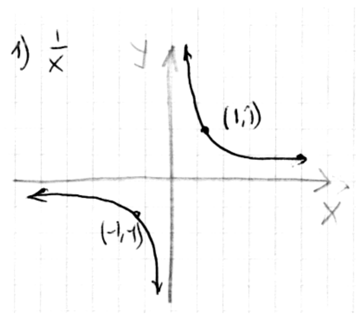
a) No, the function is increasing on interval $[0, +\infty)$.
b) Yes, the function is decreasing on $[-6, 0]$.
c) Local minimum is -9 at 0.
d) Local maximum is 0 at -4.

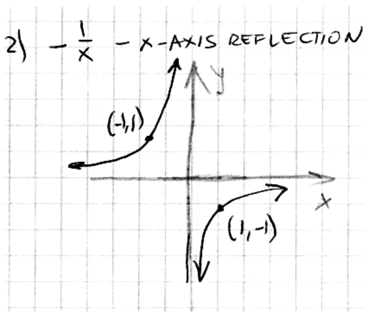
Problem 10: Graph the following function:



We have, $f(-4) = -1, f(4) = \sqrt(4) = 2$, function is increasing on $[-1, +\infty)$.

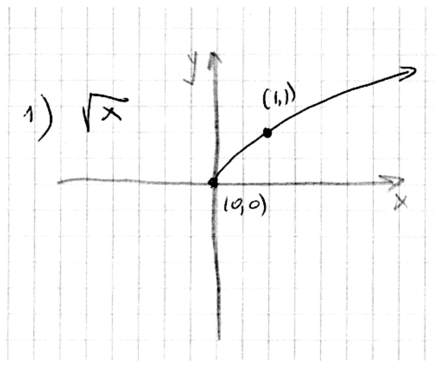
Problem 11:



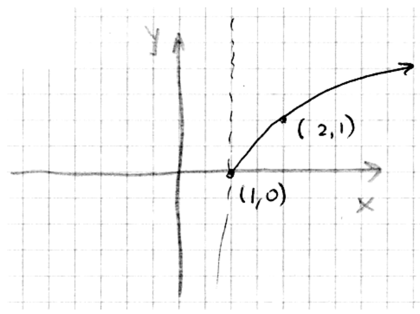


Problem 12:

We first plot the elementary function related to $g(x)$. In this case this is $\sqrt{x}$:



The next step is to move horizontally graph of $\sqrt{x}$ by 1 to obtain the graph of $\sqrt{x-1}$.



At last, we move the graph of $\sqrt{x-1}$ up by 1 to obtain $g(x) = \sqrt{x-1} + 1$.