

# Supplement to Internally 4-connected projective-planar graphs

Guoli Ding and Perry Iverson

Mathematics Department, Louisiana State University, Baton Rouge, LA 70803

July 22, 2013

This document contains the edge lists for all graphs in  $\mathcal{A}$  and  $\mathcal{A}'_4$ , as well as all of the minor-minimal graphs generated in the proofs of Lemmas 3.1–3.12.

## Graphs in $\mathcal{A}$

$A_5 = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{4, 5\}, \{6, 7\}, \{6, 8\}, \{6, 9\}, \{6, 10\}, \{7, 8\}, \{7, 9\}, \{7, 10\}, \{8, 9\}, \{8, 10\}, \{9, 10\}\}$   
 $C_{11} = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{4, 5\}, \{6, 9\}, \{6, 10\}, \{6, 11\}, \{7, 9\}, \{7, 10\}, \{7, 11\}, \{8, 9\}, \{8, 10\}, \{8, 11\}\}$   
 $E_{42} = \{\{1, 4\}, \{1, 5\}, \{1, 6\}, \{2, 4\}, \{2, 5\}, \{2, 6\}, \{3, 4\}, \{3, 5\}, \{3, 6\}, \{7, 10\}, \{7, 11\}, \{7, 12\}, \{8, 10\}, \{8, 11\}, \{8, 12\}, \{9, 10\}, \{9, 11\}, \{9, 12\}\}$   
 $A_1 = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{4, 5\}, \{5, 6\}, \{5, 7\}, \{5, 8\}, \{5, 9\}, \{6, 7\}, \{6, 8\}, \{6, 9\}, \{7, 8\}, \{7, 9\}, \{8, 9\}\}$   
 $C_1 = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{4, 5\}, \{5, 6\}, \{5, 7\}, \{5, 8\}, \{6, 9\}, \{6, 10\}, \{7, 9\}, \{7, 10\}, \{8, 9\}, \{8, 10\}\}$   
 $E_1 = \{\{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 6\}, \{4, 6\}, \{5, 6\}, \{6, 7\}, \{6, 8\}, \{6, 9\}, \{7, 10\}, \{7, 11\}, \{8, 10\}, \{8, 11\}, \{9, 10\}, \{9, 11\}\}$   
 $B_3 = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{4, 6\}, \{4, 7\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{5, 8\}, \{6, 7\}, \{6, 8\}, \{7, 8\}\}$   
 $C_2 = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{4, 6\}, \{4, 7\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{5, 8\}, \{6, 9\}, \{7, 9\}, \{8, 9\}\}$   
 $D_1 = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 5\}, \{2, 6\}, \{3, 5\}, \{3, 6\}, \{4, 5\}, \{4, 6\}, \{5, 7\}, \{5, 8\}, \{5, 9\}, \{6, 7\}, \{6, 8\}, \{6, 9\}, \{7, 10\}, \{8, 10\}, \{9, 10\}\}$   
 $D_4 = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{4, 6\}, \{4, 8\}, \{5, 7\}, \{5, 9\}, \{6, 7\}, \{6, 9\}, \{7, 8\}, \{8, 9\}\}$   
 $E_6 = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 5\}, \{2, 6\}, \{3, 5\}, \{3, 6\}, \{4, 5\}, \{4, 6\}, \{5, 7\}, \{5, 9\}, \{6, 8\}, \{6, 10\}, \{7, 9\}, \{7, 11\}, \{10, 11\}, \{10, 13\}, \{11, 12\}, \{12, 13\}\}$   
 $F_6 = \{\{1, 2\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 6\}, \{3, 4\}, \{3, 5\}, \{4, 6\}, \{5, 7\}, \{5, 9\}, \{6, 8\}, \{6, 10\}, \{7, 8\}, \{7, 10\}, \{8, 9\}, \{9, 10\}\}$   
 $B_1 = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{3, 6\}, \{3, 7\}, \{4, 5\}, \{4, 6\}, \{4, 7\}, \{5, 6\}, \{5, 7\}, \{6, 7\}\}$   
 $C_7 = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 6\}, \{4, 7\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{5, 8\}, \{6, 7\}, \{6, 8\}, \{7, 8\}\}$   
 $D_3 = \{\{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 6\}, \{4, 6\}, \{4, 7\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{5, 8\}, \{6, 7\}, \{6, 8\}, \{7, 8\}\}$   
 $D_9 = \{\{1, 3\}, \{1, 4\}, \{1, 7\}, \{2, 3\}, \{2, 4\}, \{2, 7\}, \{3, 5\}, \{3, 6\}, \{4, 5\}, \{4, 9\}, \{4, 10\}, \{5, 8\}, \{6, 7\}, \{6, 8\}, \{7, 9\}, \{7, 10\}, \{8, 9\}, \{8, 10\}\}$   
 $D_{12} = \{\{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 6\}, \{3, 9\}, \{4, 6\}, \{4, 7\}, \{4, 8\}, \{5, 7\}, \{5, 9\}, \{6, 7\}, \{6, 8\}, \{7, 8\}, \{8, 9\}\}$   
 $E_3 = \{\{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 6\}, \{3, 7\}, \{3, 8\}, \{4, 6\}, \{4, 7\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{5, 8\}\}$   
 $E_5 = \{\{1, 3\}, \{1, 4\}, \{1, 6\}, \{2, 3\}, \{2, 4\}, \{2, 6\}, \{3, 5\}, \{4, 5\}, \{4, 8\}, \{4, 9\}, \{5, 6\}, \{5, 7\}, \{6, 8\}, \{6, 9\}, \{7, 8\}, \{7, 9\}\}$   
 $E_{11} = \{\{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 6\}, \{3, 7\}, \{4, 6\}, \{4, 8\}, \{4, 9\}, \{5, 7\}, \{5, 8\}, \{6, 10\}, \{7, 9\}, \{8, 10\}, \{9, 10\}\}$   
 $E_{19} = \{\{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 6\}, \{4, 7\}, \{5, 8\}, \{5, 9\}, \{6, 7\}, \{6, 8\}, \{6, 9\}, \{7, 8\}, \{7, 9\}, \{8, 9\}\}$   
 $E_{27} = \{\{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 7\}, \{3, 8\}, \{4, 6\}, \{4, 10\}, \{5, 8\}, \{5, 9\}, \{6, 7\}, \{6, 9\}, \{7, 10\}, \{8, 10\}, \{9, 10\}\}$   
 $F_1 = \{\{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 6\}, \{4, 6\}, \{4, 7\}, \{5, 8\}, \{5, 9\}, \{6, 8\}, \{6, 9\}, \{7, 8\}, \{7, 9\}\}$   
 $G_1 = \{\{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 6\}, \{4, 7\}, \{5, 8\}, \{6, 9\}, \{6, 10\}, \{7, 9\}, \{7, 10\}, \{8, 9\}, \{8, 10\}\}$

# Graphs in $\mathcal{A}'_4$

$$\begin{aligned}
A_2 &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{1, 6\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{2, 7\}, \{3, 4\}, \{3, 6\}, \{3, 7\}, \{4, 5\}, \{4, 6\}, \{4, 7\}, \{5, 6\}, \{5, 7\}, \{6, 7\}\} \\
B_7 &= \{\{1, 2\}, \{1, 3\}, \{1, 5\}, \{1, 6\}, \{2, 4\}, \{2, 5\}, \{2, 6\}, \{2, 7\}, \{3, 4\}, \{3, 8\}, \{4, 5\}, \{4, 7\}, \{5, 6\}, \{5, 7\}, \{5, 8\}, \{6, 7\}, \{6, 8\}, \{7, 8\}\} \\
C_3 &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 5\}, \{2, 6\}, \{2, 9\}, \{3, 5\}, \{3, 6\}, \{3, 7\}, \{4, 5\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{5, 9\}, \{6, 7\}, \{6, 9\}, \{7, 8\}, \{8, 9\}\} \\
C_4 &= \{\{1, 2\}, \{1, 3\}, \{1, 5\}, \{1, 6\}, \{2, 4\}, \{2, 5\}, \{2, 7\}, \{3, 4\}, \{3, 8\}, \{4, 5\}, \{4, 9\}, \{5, 6\}, \{5, 8\}, \{5, 9\}, \{6, 7\}, \{6, 8\}, \{7, 9\}, \{8, 9\}\} \\
D_2 &= \{\{1, 2\}, \{1, 4\}, \{1, 6\}, \{2, 3\}, \{2, 7\}, \{3, 5\}, \{3, 6\}, \{3, 8\}, \{4, 5\}, \{4, 9\}, \{5, 6\}, \{5, 8\}, \{6, 7\}, \{6, 8\}, \{6, 9\}, \{7, 10\}, \{8, 10\}, \{9, 10\}\} \\
D_{17} &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 7\}, \{2, 3\}, \{2, 4\}, \{2, 8\}, \{3, 4\}, \{3, 5\}, \{4, 6\}, \{5, 6\}, \{5, 7\}, \{5, 8\}, \{6, 7\}, \{6, 8\}, \{7, 8\}\} \\
E_2 &= \{\{1, 2\}, \{1, 3\}, \{1, 8\}, \{2, 4\}, \{2, 7\}, \{3, 6\}, \{3, 7\}, \{4, 5\}, \{4, 9\}, \{5, 6\}, \{5, 7\}, \{6, 10\}, \{7, 8\}, \{7, 9\}, \{7, 10\}, \{8, 11\}, \{9, 11\}, \{10, 11\}\} \\
E_{18} &= \{\{1, 2\}, \{1, 3\}, \{1, 5\}, \{1, 8\}, \{2, 4\}, \{2, 6\}, \{2, 7\}, \{3, 4\}, \{3, 7\}, \{4, 5\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{6, 8\}, \{7, 8\}\} \\
E_{20} &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 7\}, \{2, 3\}, \{2, 4\}, \{2, 6\}, \{3, 4\}, \{3, 8\}, \{4, 5\}, \{5, 6\}, \{5, 7\}, \{5, 8\}, \{6, 9\}, \{7, 9\}, \{8, 9\}\} \\
E_{22} &= \{\{1, 2\}, \{1, 3\}, \{1, 5\}, \{1, 8\}, \{2, 4\}, \{2, 9\}, \{3, 4\}, \{3, 6\}, \{4, 5\}, \{4, 7\}, \{5, 6\}, \{5, 9\}, \{6, 7\}, \{6, 8\}, \{7, 9\}, \{8, 9\}\} \\
F_4 &= \{\{1, 2\}, \{1, 4\}, \{1, 8\}, \{2, 3\}, \{2, 7\}, \{3, 5\}, \{3, 6\}, \{3, 10\}, \{4, 5\}, \{4, 9\}, \{5, 8\}, \{6, 7\}, \{6, 8\}, \{7, 10\}, \{8, 9\}, \{9, 10\}\} \\
B'_1 &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{1, 6\}, \{1, 7\}, \{2, 3\}, \{2, 4\}, \{2, 6\}, \{3, 4\}, \{3, 5\}, \{3, 6\}, \{4, 5\}, \{4, 6\}, \{4, 7\}, \{5, 6\}, \{5, 7\}, \{6, 7\}\} \\
B''_1 &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 7\}, \{2, 3\}, \{2, 5\}, \{2, 6\}, \{2, 7\}, \{2, 8\}, \{3, 5\}, \{3, 7\}, \{4, 5\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{6, 7\}, \{6, 8\}, \{7, 8\}\} \\
B'''_1 &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 4\}, \{2, 5\}, \{2, 6\}, \{3, 4\}, \{3, 6\}, \{3, 7\}, \{4, 5\}, \{4, 7\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{5, 8\}, \{6, 8\}, \{7, 8\}\} \\
D'_3 &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{1, 7\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{3, 6\}, \{4, 7\}, \{4, 8\}, \{5, 7\}, \{5, 8\}, \{6, 7\}, \{6, 8\}\} \\
D''_3 &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 6\}, \{2, 3\}, \{2, 5\}, \{2, 6\}, \{3, 5\}, \{3, 6\}, \{3, 7\}, \{4, 5\}, \{4, 8\}, \{5, 9\}, \{6, 8\}, \{6, 9\}, \{7, 8\}, \{7, 9\}\} \\
E'_3 &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 6\}, \{1, 8\}, \{2, 3\}, \{2, 5\}, \{2, 7\}, \{3, 5\}, \{3, 6\}, \{3, 7\}, \{4, 5\}, \{4, 7\}, \{5, 6\}, \{5, 8\}, \{6, 7\}, \{6, 8\}, \{7, 8\}\} \\
E''_3 &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 7\}, \{1, 9\}, \{2, 3\}, \{2, 5\}, \{2, 8\}, \{3, 6\}, \{3, 8\}, \{4, 5\}, \{4, 8\}, \{5, 6\}, \{5, 9\}, \{6, 7\}, \{7, 8\}, \{7, 9\}, \{8, 9\}\} \\
E''_6 &= \{\{1, 2\}, \{1, 4\}, \{1, 6\}, \{1, 7\}, \{2, 3\}, \{2, 5\}, \{3, 4\}, \{3, 6\}, \{4, 5\}, \{4, 7\}, \{4, 9\}, \{5, 6\}, \{5, 8\}, \{6, 7\}, \{6, 9\}, \{7, 8\}, \{8, 9\}\} \\
E''_9 &= \{\{1, 2\}, \{1, 4\}, \{1, 7\}, \{2, 3\}, \{2, 6\}, \{3, 5\}, \{3, 7\}, \{4, 5\}, \{4, 8\}, \{5, 6\}, \{5, 10\}, \{6, 7\}, \{6, 9\}, \{7, 8\}, \{7, 10\}, \{8, 9\}, \{9, 10\}\} \\
F'_1 &= \{\{1, 2\}, \{1, 4\}, \{1, 5\}, \{1, 8\}, \{2, 3\}, \{2, 7\}, \{2, 9\}, \{3, 4\}, \{3, 5\}, \{4, 9\}, \{5, 6\}, \{5, 8\}, \{6, 7\}, \{6, 9\}, \{7, 8\}, \{8, 9\}\} \\
F''_1 &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 5\}, \{2, 7\}, \{3, 6\}, \{3, 7\}, \{4, 7\}, \{4, 8\}, \{4, 9\}, \{5, 6\}, \{5, 8\}, \{6, 9\}, \{7, 10\}, \{8, 10\}, \{9, 10\}\} \\
G'_1 &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 5\}, \{2, 7\}, \{3, 5\}, \{3, 8\}, \{4, 5\}, \{4, 9\}, \{5, 6\}, \{6, 7\}, \{6, 8\}, \{6, 9\}, \{7, 10\}, \{8, 10\}, \{9, 10\}\}
\end{aligned}$$

## Auxiliary Graphs

$$\begin{aligned}
B_1^a &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 6\}, \{3, 7\}, \{3, 8\}, \{4, 6\}, \{4, 7\}, \{4, 8\}, \{5, 6\}, \{5, 8\}, \{6, 7\}, \{7, 8\}\} \\
D_3^a &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 7\}, \{3, 8\}, \{4, 5\}, \{4, 6\}, \{4, 8\}, \{5, 7\}, \{5, 8\}, \{6, 7\}, \{6, 8\}\} \\
D_3^{ga} &= \{\{1, 2\}, \{1, 3\}, \{1, 5\}, \{2, 4\}, \{2, 6\}, \{3, 4\}, \{3, 5\}, \{3, 6\}, \{4, 7\}, \{4, 8\}, \{4, 9\}, \{5, 7\}, \{5, 8\}, \{6, 7\}, \{6, 8\}, \{6, 9\}, \{7, 9\}, \{8, 9\}\} \\
D_3^b &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 5\}, \{3, 6\}, \{3, 8\}, \{4, 6\}, \{4, 7\}, \{5, 7\}, \{5, 9\}, \{6, 9\}, \{7, 8\}, \{8, 9\}\} \\
E_3^a &= \{\{1, 4\}, \{1, 5\}, \{1, 6\}, \{1, 7\}, \{1, 8\}, \{2, 4\}, \{2, 5\}, \{2, 6\}, \{2, 7\}, \{2, 8\}, \{3, 4\}, \{3, 5\}, \{3, 6\}, \{3, 7\}, \{3, 8\}, \{4, 6\}\} \\
E_3^{ga} &= \{\{1, 4\}, \{1, 5\}, \{1, 6\}, \{1, 7\}, \{1, 8\}, \{2, 4\}, \{2, 5\}, \{2, 6\}, \{2, 7\}, \{2, 8\}, \{3, 4\}, \{3, 5\}, \{3, 6\}, \{3, 7\}, \{3, 8\}, \{4, 5\}, \{4, 6\}\} \\
E_3^b &= \{\{1, 5\}, \{1, 6\}, \{1, 8\}, \{1, 9\}, \{2, 4\}, \{2, 5\}, \{2, 7\}, \{2, 8\}, \{2, 9\}, \{3, 4\}, \{3, 5\}, \{3, 7\}, \{3, 8\}, \{3, 9\}, \{4, 6\}, \{6, 7\}\} \\
E_3^{ba} &= \{\{1, 4\}, \{1, 5\}, \{1, 6\}, \{2, 4\}, \{2, 5\}, \{2, 7\}, \{2, 8\}, \{2, 9\}, \{3, 4\}, \{3, 5\}, \{3, 7\}, \{3, 8\}, \{3, 9\}, \{4, 7\}, \{6, 7\}, \{6, 8\}, \{6, 9\}\} \\
E_3^{bb} &= \{\{1, 4\}, \{1, 5\}, \{1, 7\}, \{2, 5\}, \{2, 6\}, \{2, 9\}, \{2, 10\}, \{3, 4\}, \{3, 5\}, \{3, 8\}, \{3, 9\}, \{3, 10\}, \{4, 6\}, \{6, 8\}, \{7, 8\}, \{7, 9\}, \{7, 10\}\} \\
E_5^a &= \{\{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 6\}, \{3, 7\}, \{3, 8\}, \{4, 6\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{5, 8\}, \{6, 9\}, \{7, 9\}, \{8, 9\}\} \\
E_5^b &= \{\{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 6\}, \{3, 7\}, \{3, 9\}, \{4, 6\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{5, 9\}, \{6, 7\}, \{7, 8\}, \{8, 9\}\} \\
F_1^a &= \{\{1, 4\}, \{1, 5\}, \{1, 7\}, \{1, 8\}, \{2, 4\}, \{2, 5\}, \{2, 6\}, \{3, 4\}, \{3, 5\}, \{3, 6\}, \{3, 9\}, \{4, 9\}, \{6, 7\}, \{6, 8\}, \{7, 9\}, \{8, 9\}\} \\
F_1^b &= \{\{1, 4\}, \{1, 5\}, \{1, 7\}, \{2, 4\}, \{2, 5\}, \{2, 10\}, \{3, 4\}, \{3, 5\}, \{3, 7\}, \{3, 10\}, \{4, 6\}, \{6, 8\}, \{6, 9\}, \{7, 8\}, \{7, 9\}, \{8, 10\}, \{9, 10\}\} \\
F_1^c &= \{\{1, 4\}, \{1, 5\}, \{1, 8\}, \{1, 9\}, \{2, 4\}, \{2, 5\}, \{2, 7\}, \{3, 5\}, \{3, 6\}, \{3, 10\}, \{4, 6\}, \{6, 7\}, \{7, 8\}, \{7, 9\}, \{8, 10\}, \{9, 10\}\} \\
F_1^d &= \{\{1, 4\}, \{1, 5\}, \{1, 7\}, \{2, 4\}, \{2, 5\}, \{2, 11\}, \{3, 4\}, \{3, 5\}, \{3, 10\}, \{3, 11\}, \{4, 6\}, \{6, 8\}, \{6, 9\}, \{7, 8\}, \{7, 9\}, \{7, 11\}, \{8, 10\}, \{9, 10\}\}
\end{aligned}$$

## $B_1$ Minor-Minimal Twists

- 1:  $\{\{1, 4\}, \{1, 5\}, \{1, 6\}, \{1, 7\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{2, 6\}, \{2, 7\}, \{3, 4\}, \{3, 5\}, \{3, 6\}, \{4, 5\}, \{4, 6\}, \{4, 7\}, \{5, 6\}, \{5, 7\}, \{6, 7\}\}$
- 2:  $\{\{1, 2\}, \{1, 7\}, \{1, 8\}, \{1, 9\}, \{2, 4\}, \{2, 9\}, \{3, 4\}, \{3, 5\}, \{3, 6\}, \{3, 7\}, \{3, 9\}, \{4, 5\}, \{4, 6\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{6, 7\}, \{7, 8\}, \{7, 9\}, \{8, 9\}\}$
- 3:  $\{\{1, 2\}, \{1, 7\}, \{1, 8\}, \{1, 9\}, \{2, 4\}, \{2, 9\}, \{3, 5\}, \{3, 6\}, \{3, 7\}, \{3, 8\}, \{3, 9\}, \{4, 7\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{5, 8\}, \{6, 7\}, \{6, 8\}, \{7, 9\}, \{8, 9\}\}$
- 4:  $\{\{1, 2\}, \{1, 7\}, \{1, 8\}, \{1, 9\}, \{2, 4\}, \{2, 9\}, \{3, 5\}, \{3, 6\}, \{3, 7\}, \{3, 8\}, \{3, 9\}, \{4, 5\}, \{4, 6\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{6, 7\}, \{7, 8\}, \{7, 9\}, \{8, 9\}\}$
- 5:  $\{\{1, 2\}, \{1, 7\}, \{1, 8\}, \{1, 9\}, \{2, 4\}, \{2, 9\}, \{3, 4\}, \{3, 5\}, \{3, 6\}, \{3, 7\}, \{3, 9\}, \{4, 7\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{5, 8\}, \{6, 7\}, \{6, 8\}, \{7, 9\}, \{8, 9\}\}$
- 6:  $\{\{1, 2\}, \{1, 7\}, \{1, 8\}, \{1, 9\}, \{2, 4\}, \{2, 9\}, \{3, 5\}, \{3, 6\}, \{3, 7\}, \{3, 8\}, \{3, 9\}, \{4, 6\}, \{4, 7\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{5, 8\}, \{6, 7\}, \{7, 9\}\}$

















## $F_1^c$ Minor-Minimal Twists

- 1 :  $\{\{1, 6\}, \{1, 7\}, \{1, 9\}, \{1, 10\}, \{2, 4\}, \{2, 5\}, \{2, 9\}, \{2, 10\}, \{3, 5\}, \{3, 7\}, \{3, 8\}, \{4, 6\}, \{4, 7\}, \{4, 9\}, \{5, 6\}, \{8, 9\}, \{8, 10\}\}$
- 2 :  $\{\{1, 6\}, \{1, 7\}, \{1, 9\}, \{1, 10\}, \{2, 4\}, \{2, 5\}, \{2, 9\}, \{2, 10\}, \{3, 5\}, \{3, 7\}, \{3, 9\}, \{4, 6\}, \{4, 7\}, \{4, 8\}, \{5, 6\}, \{8, 9\}, \{8, 10\}\}$
- 3 :  $\{\{1, 6\}, \{1, 7\}, \{1, 10\}, \{1, 11\}, \{2, 4\}, \{2, 5\}, \{2, 10\}, \{3, 5\}, \{3, 7\}, \{3, 8\}, \{4, 6\}, \{4, 7\}, \{4, 9\}, \{5, 6\}, \{8, 10\}, \{8, 11\}, \{9, 10\}, \{9, 11\}\}$
- 4 :  $\{\{1, 6\}, \{1, 7\}, \{1, 9\}, \{1, 10\}, \{2, 4\}, \{2, 5\}, \{2, 9\}, \{2, 10\}, \{3, 5\}, \{3, 7\}, \{3, 8\}, \{4, 6\}, \{4, 7\}, \{5, 6\}, \{5, 9\}, \{8, 9\}, \{8, 10\}\}$
- 5 :  $\{\{1, 6\}, \{1, 7\}, \{1, 9\}, \{1, 10\}, \{2, 4\}, \{2, 5\}, \{2, 9\}, \{2, 10\}, \{3, 5\}, \{3, 7\}, \{3, 9\}, \{4, 6\}, \{4, 7\}, \{5, 6\}, \{5, 8\}, \{8, 9\}, \{8, 10\}\}$
- 6 :  $\{\{1, 6\}, \{1, 7\}, \{1, 10\}, \{1, 11\}, \{2, 4\}, \{2, 5\}, \{2, 10\}, \{3, 5\}, \{3, 7\}, \{3, 8\}, \{4, 6\}, \{4, 7\}, \{5, 6\}, \{5, 9\}, \{8, 10\}, \{8, 11\}, \{9, 10\}, \{9, 11\}\}$
- 7 :  $\{\{1, 6\}, \{1, 7\}, \{1, 10\}, \{2, 4\}, \{2, 5\}, \{2, 10\}, \{2, 11\}, \{3, 5\}, \{3, 7\}, \{3, 8\}, \{4, 6\}, \{4, 7\}, \{5, 6\}, \{5, 9\}, \{8, 10\}, \{8, 11\}, \{9, 10\}, \{9, 11\}\}$
- 8 :  $\{\{1, 7\}, \{1, 8\}, \{1, 11\}, \{2, 4\}, \{2, 5\}, \{2, 10\}, \{2, 11\}, \{3, 5\}, \{3, 7\}, \{3, 9\}, \{4, 6\}, \{4, 7\}, \{5, 6\}, \{6, 8\}, \{8, 10\}, \{9, 10\}, \{9, 11\}\}$

## $F_1^d$ Minor-Minimal Twists

- 1 :  $\{\{1, 3\}, \{1, 4\}, \{1, 6\}, \{1, 8\}, \{2, 4\}, \{2, 7\}, \{2, 10\}, \{2, 11\}, \{3, 5\}, \{3, 7\}, \{3, 9\}, \{4, 5\}, \{4, 10\}, \{5, 6\}, \{6, 7\}, \{8, 10\}, \{8, 11\}, \{9, 10\}, \{9, 11\}\}$
- 2 :  $\{\{1, 3\}, \{1, 4\}, \{1, 6\}, \{1, 8\}, \{2, 4\}, \{2, 7\}, \{2, 10\}, \{2, 11\}, \{3, 5\}, \{3, 7\}, \{3, 10\}, \{4, 5\}, \{4, 9\}, \{5, 6\}, \{6, 7\}, \{8, 10\}, \{8, 11\}, \{9, 10\}, \{9, 11\}\}$
- 3 :  $\{\{1, 3\}, \{1, 4\}, \{1, 6\}, \{1, 8\}, \{2, 4\}, \{2, 7\}, \{2, 11\}, \{3, 5\}, \{3, 7\}, \{3, 9\}, \{4, 5\}, \{4, 10\}, \{5, 6\}, \{6, 7\}, \{8, 11\}, \{8, 12\}, \{9, 11\}, \{9, 12\}, \{10, 11\}, \{10, 12\}\}$
- 4 :  $\{\{1, 3\}, \{1, 4\}, \{1, 6\}, \{1, 10\}, \{2, 4\}, \{2, 7\}, \{2, 10\}, \{2, 11\}, \{3, 5\}, \{3, 7\}, \{3, 8\}, \{4, 5\}, \{4, 9\}, \{5, 6\}, \{6, 7\}, \{8, 10\}, \{8, 11\}, \{9, 10\}, \{9, 11\}\}$
- 5 :  $\{\{1, 3\}, \{1, 4\}, \{1, 6\}, \{1, 8\}, \{2, 4\}, \{2, 7\}, \{2, 10\}, \{2, 11\}, \{3, 5\}, \{3, 7\}, \{3, 9\}, \{4, 5\}, \{5, 6\}, \{5, 10\}, \{6, 7\}, \{8, 10\}, \{8, 11\}, \{9, 10\}, \{9, 11\}\}$
- 6 :  $\{\{1, 3\}, \{1, 4\}, \{1, 6\}, \{1, 8\}, \{2, 4\}, \{2, 7\}, \{2, 10\}, \{2, 11\}, \{3, 5\}, \{3, 7\}, \{3, 10\}, \{4, 5\}, \{5, 6\}, \{5, 9\}, \{6, 7\}, \{8, 10\}, \{8, 11\}, \{9, 10\}, \{9, 11\}\}$
- 7 :  $\{\{1, 3\}, \{1, 4\}, \{1, 6\}, \{1, 8\}, \{2, 4\}, \{2, 7\}, \{2, 11\}, \{3, 5\}, \{3, 7\}, \{3, 9\}, \{4, 5\}, \{5, 6\}, \{5, 10\}, \{6, 7\}, \{8, 11\}, \{8, 12\}, \{9, 11\}, \{9, 12\}, \{10, 11\}, \{10, 12\}\}$
- 8 :  $\{\{1, 3\}, \{1, 4\}, \{1, 6\}, \{1, 9\}, \{2, 7\}, \{2, 8\}, \{2, 12\}, \{3, 5\}, \{3, 7\}, \{3, 10\}, \{4, 5\}, \{4, 8\}, \{5, 6\}, \{6, 7\}, \{8, 11\}, \{9, 11\}, \{9, 12\}, \{10, 11\}, \{10, 12\}\}$
- 9 :  $\{\{1, 4\}, \{1, 6\}, \{1, 8\}, \{1, 9\}, \{2, 4\}, \{2, 7\}, \{2, 11\}, \{2, 12\}, \{3, 5\}, \{3, 7\}, \{3, 8\}, \{3, 10\}, \{4, 5\}, \{5, 6\}, \{6, 7\}, \{8, 11\}, \{9, 11\}, \{9, 12\}, \{10, 11\}, \{10, 12\}\}$
- 10 :  $\{\{1, 4\}, \{1, 6\}, \{1, 8\}, \{1, 9\}, \{2, 4\}, \{2, 7\}, \{2, 11\}, \{2, 12\}, \{3, 5\}, \{3, 7\}, \{3, 8\}, \{3, 11\}, \{4, 5\}, \{5, 6\}, \{6, 7\}, \{8, 10\}, \{9, 11\}, \{9, 12\}, \{10, 11\}, \{10, 12\}\}$
- 11 :  $\{\{1, 4\}, \{1, 6\}, \{1, 8\}, \{1, 9\}, \{2, 4\}, \{2, 7\}, \{2, 12\}, \{3, 5\}, \{3, 7\}, \{3, 8\}, \{3, 10\}, \{4, 5\}, \{5, 6\}, \{6, 7\}, \{8, 11\}, \{9, 12\}, \{9, 13\}, \{10, 12\}, \{10, 13\}, \{11, 12\}, \{11, 13\}\}$