

## Excluding a small minor (supplement)

In our proofs of Theorem 4.5 and Theorem 4.6, some assertions are verified using a computer. The purpose of this note is to provide some details on these computations.

Our program, written in *Mathematica*, performs the following computations:

1. for any input graphs  $G$  and  $H$ , determine if  $G$  and  $H$  are isomorphic;
2. for any input graphs  $G$  and  $H$ , determine if  $G$  is  $H$ -free;
3. for any input graph  $G$ , generate all undeltions and uncontractions of  $G$ .

For any two sets  $\mathbf{X}$ ,  $\mathbf{Y}$  of graphs, let  $\mathbf{grow}[\mathbf{X}, \mathbf{Y}] = \{G: G \text{ is an undeletion or uncontraction of some } x \in \mathbf{X} \text{ such that } G \text{ is } y\text{-free for all } y \in \mathbf{Y}\}$ . It is clear that  $\mathbf{grow}[\mathbf{X}, \mathbf{Y}]$  can be computed using the above programs. We remark that, almost all we need from a computer is to compute  $\mathbf{grow}[\mathbf{X}, \mathbf{Y}]$  for different choices of  $\mathbf{X}$  and  $\mathbf{Y}$ .

### Computations in proving Theorem 4.5.

The following assertion is made in the proof: Let  $\mathcal{S}$  be defined as in the paper. Let  $\mathcal{S}_0 \subseteq \mathcal{S}$  consist of graphs  $H$  such that  $H$  is a 3-sum of  $H_1$  and  $H_2$ , each of which is either a Prism or a wheel on at most six vertices, and  $H$  contains  $W_5+e$  as a minor. If  $G$  is an undeletion or an uncontraction of any  $H \in \mathcal{S}_0$  and if  $G$  is Octahedron\( $e$ -free, then  $G$  belongs to  $\mathcal{S}$ . Since all graphs in  $\mathcal{S}_0$  can be obtained from  $W_5+e$  by undeletions and uncontractions, this assertion can be verified as follows.

- Let  $\mathbf{Max}$  consist of the four maximal graphs in  $\mathcal{S}_0$ , each which is a 3-sum of two graphs from  $\{\text{Prism}, W_5\}$ . Then we can test if a graph is in  $\mathcal{S}_0$  by testing if it is a minor of a graph in  $\mathbf{Max}$ .
- Let  $\mathbf{g}_{11} = \{W_5+e\}$ , which consists of all graphs in  $\mathcal{S}_0$  on (at most) 11 edges.
- Inductively, if  $\mathbf{g}_k$ , the set of  $k$ -edge graphs in  $\mathcal{S}_0$  has been determined, we perform the following operations:
  1. we compute  $\mathbf{h}_{k+1} = \mathbf{grow}[\mathbf{g}_k, \{\text{Octahedron}\setminus e\}]$
  2. we compute  $\mathbf{g}_{k+1} = \mathbf{h}_{k+1} \cap \mathcal{S}_0$  by using  $\mathbf{Max}$
  3. we verify (by hand) that  $\mathbf{h}_{k+1} \setminus \mathbf{g}_{k+1} \subseteq \mathcal{S}$ .

In the following we list all sets  $\mathbf{g}_k$  and  $\mathbf{h}_k$ , which should help the reader to confirm our computation.

$\mathbf{h}_{12} = \{$

FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}}],  
FromUnorderedPairs[{{1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {1, 3}, {2, 7}, {5, 7}, {1, 7}}],  
FromUnorderedPairs[{{1, 5}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {1, 3}, {2, 7}, {6, 7}, {1, 7}}],  
FromUnorderedPairs[{{1, 2}, {1, 5}, {2, 3}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {1, 3}, {1, 7}, {2, 7}, {6, 7}}],  
FromUnorderedPairs[{{1, 2}, {1, 5}, {2, 3}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {1, 3}, {1, 7}, {3, 7}, {6, 7}}],  
FromUnorderedPairs[{{1, 2}, {1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {1, 3}, {4, 7}, {5, 7}, {6, 7}}];

$g_{12} = h_{12} [\{1, 3, 4, 6\}]$  (this is the restriction of  $h_{12}$  to the 1<sup>st</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, and 6<sup>th</sup> of its members).

$h_{13} = \{$

FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {1, 3}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {6, 7}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {2, 8}, {4, 8}, {3, 8}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {2, 8}, {6, 8}, {3, 8}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {1, 7}, {2, 8}, {7, 8}, {3, 8}}],  
FromUnorderedPairs[{{1, 5}, {2, 3}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {1, 8}, {3, 8}, {6, 8}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {2, 8}, {3, 8}, {6, 8}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {2, 7}, {3, 7}, {1, 7}, {4, 8}, {5, 8}, {6, 8}}],  
FromUnorderedPairs[{{1, 5}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {1, 3}, {2, 7}, {6, 7}, {1, 7}, {1, 6}}],  
FromUnorderedPairs[{{1, 5}, {2, 3}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {6, 7}, {1, 7}, {1, 8}, {6, 8}, {3, 8}}],  
FromUnorderedPairs[{{1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {1, 3}, {4, 7}, {5, 7}, {6, 7}, {2, 8}, {5, 8}, {1, 8}}];

$g_{13} = h_{13} [ \{1, 2, 4, 5, 7, 8, 9\} ]$

$h_{14} = \{$

FromUnorderedPairs[{{1, 5}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {3, 8}, {6, 8}, {1, 8}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {1, 8}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {1, 8}, {6, 8}, {3, 8}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {1, 7}, {1, 3}, {2, 8}, {7, 8}, {3, 8}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {1, 3}, {4, 8}, {6, 8}, {3, 8}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {6, 7}, {2, 8}, {4, 8}, {3, 8}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {6, 7}, {2, 8}, {6, 8}, {3, 8}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {1, 7}, {6, 7}, {2, 8}, {7, 8}, {3, 8}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {3, 8}, {7, 8}, {6, 8}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {2, 8}, {3, 8}, {2, 9}, {8, 9}, {6, 9}}],  
FromUnorderedPairs[{{1, 5}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {1, 7}, {2, 8}, {7, 8}, {3, 8}, {1, 9}, {3, 9}, {6, 9}}],  
FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {2, 7}, {1, 7}, {2, 8}, {7, 8}, {3, 8}, {4, 9}, {5, 9}, {6, 9}}],  
FromUnorderedPairs[{{1, 5}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {6, 7}, {1, 7}, {3, 8}, {6, 8}, {1, 8}}],  
FromUnorderedPairs[{{1, 5}, {2, 3}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {1, 3}, {2, 7}, {6, 7}, {1, 7}, {1, 8}, {3, 8}, {6, 8}}];

$g_{14} = h_{14} [ \{2, 4, 5, 7, 8, 10, 12\} ]$

$h_{15} = \{$

FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {1, 8}, {1, 3}}],

FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 8}, {7, 8}, {1, 8}, {2, 9}, {7, 9}, {3, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {1, 8}, {4, 9}, {6, 9}, {3, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {7, 8}, {1, 8}, {6, 9}, {8, 9}, {3, 9}}],  
 FromUnorderedPairs[{{1, 5}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {1, 7}, {2, 8}, {7, 8}, {3, 8}, {3, 9}, {6, 9}, {1, 9}}],  
 FromUnorderedPairs[{{1, 5}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {1, 7}, {1, 3}, {2, 8}, {7, 8}, {3, 8}, {1, 9}, {3, 9}, {6, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {2, 7}, {1, 7}, {1, 3}, {2, 8}, {7, 8}, {3, 8}, {4, 9}, {5, 9}, {6, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {6, 7}, {2, 8}, {6, 8}, {3, 8}, {3, 6}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {3, 7}, {6, 7}, {2, 8}, {6, 8}, {3, 8}, {1, 9}, {2, 9}, {7, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {6, 7}, {2, 8}, {6, 8}, {3, 8}, {1, 9}, {3, 9}, {7, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {2, 8}, {6, 8}, {3, 8}, {1, 9}, {6, 9}, {7, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {1, 7}, {6, 7}, {2, 8}, {7, 8}, {3, 8}, {6, 8}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {6, 7}, {2, 8}, {7, 8}, {3, 8}, {1, 9}, {2, 9}, {7, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {2, 8}, {7, 8}, {3, 8}, {1, 9}, {6, 9}, {7, 9}}],  
 FromUnorderedPairs[{{1, 5}, {2, 6}, {3, 4}, {4, 5}, {2, 7}, {1, 7}, {2, 8}, {7, 8}, {3, 8}, {4, 9}, {5, 9}, {6, 9}, {1, 10}, {3, 10}, {6, 10}}];

$\mathbf{g}_{15} = \mathbf{h}_{15}[\{1, 2, 3, 7, 8, 10, 14\}]$

$\mathbf{h}_{16} = \{$

FromUnorderedPairs[{{1, 5}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {1, 8}, {3, 9}, {6, 9}, {1, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {3, 9}, {8, 9}, {1, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {1, 8}, {1, 9}, {6, 9}, {3, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {1, 8}, {1, 3}, {4, 9}, {6, 9}, {3, 9}}],  
 FromUnorderedPairs[{{1, 5}, {2, 3}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {1, 8}, {1, 3}, {1, 9}, {3, 9}, {6, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {1, 8}, {1, 3}, {2, 9}, {3, 9}, {6, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {1, 8}, {1, 3}, {4, 9}, {5, 9}, {6, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 8}, {7, 8}, {1, 8}, {2, 9}, {7, 9}, {3, 9}, {4, 10}, {6, 10}, {3, 10}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 8}, {7, 8}, {1, 8}, {2, 9}, {7, 9}, {4, 10}, {9, 10}, {3, 10}}],  
 FromUnorderedPairs[{{1, 5}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {2, 7}, {1, 7}, {2, 8}, {7, 8}, {3, 8}, {4, 9}, {5, 9}, {6, 9}, {3, 10}, {6, 10}, {1, 10}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {6, 7}, {2, 8}, {6, 8}, {3, 8}, {4, 9}, {6, 9}, {3, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {1, 7}, {6, 7}, {2, 8}, {6, 8}, {3, 8}, {3, 6}, {4, 9}, {7, 9}, {3, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {6, 7}, {2, 8}, {6, 8}, {3, 6}, {4, 9}, {8, 9}, {3, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {1, 7}, {2, 8}, {6, 8}, {3, 8}, {3, 9}, {7, 9}, {6, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {2, 8}, {7, 8}, {3, 8}, {1, 9}, {6, 9}, {7, 9}, {6, 7}}];

$\mathbf{g}_{16} = \mathbf{h}_{16}[\{2, 4, 7, 12\}]$

$\mathbf{h}_{17} = \{$

FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {3, 9}, {8, 9}, {1, 9}, {1, 3}}],

FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {3, 9}, {8, 9}, {1, 9}, {6, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {3, 9}, {8, 9}, {1, 9}, {4, 10}, {6, 10}, {3, 10}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {8, 9}, {1, 9}, {6, 10}, {9, 10}, {3, 10}}],  
 FromUnorderedPairs[{{1, 5}, {2, 3}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {3, 9}, {8, 9}, {1, 9}, {1, 10}, {3, 10}, {6, 10}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {3, 9}, {8, 9}, {1, 9}, {2, 10}, {3, 10}, {6, 10}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {3, 9}, {8, 9}, {1, 9}, {4, 10}, {5, 10}, {6, 10}}],  
 FromUnorderedPairs[{{1, 5}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {1, 8}, {4, 9}, {5, 9}, {6, 9}, {3, 10}, {6, 10}, {1, 10}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {4, 5}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {1, 8}, {4, 9}, {5, 9}, {6, 9}, {1, 10}, {6, 10}, {3, 10}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {1, 7}, {6, 7}, {2, 8}, {6, 8}, {3, 8}, {3, 6}, {4, 9}, {7, 9}, {3, 9}, {6, 9}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {4, 5}, {4, 6}, {5, 6}, {6, 7}, {2, 8}, {6, 8}, {3, 8}, {3, 6}, {4, 9}, {7, 9}, {3, 9}, {1, 10}, {2, 10}, {7, 10}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {2, 8}, {6, 8}, {3, 8}, {3, 6}, {4, 9}, {7, 9}, {3, 9}, {1, 10}, {6, 10}, {7, 10}}];

$g_{17} = h_{17}[\{2, 10\}]$

$h_{18} = \{$

FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {3, 9}, {8, 9}, {1, 9}, {6, 9}, {4, 10}, {6, 10}, {3, 10}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {8, 9}, {1, 9}, {6, 9}, {6, 10}, {9, 10}, {3, 10}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {3, 9}, {8, 9}, {1, 10}, {6, 10}, {9, 10}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 3}, {2, 6}, {3, 4}, {3, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {3, 7}, {3, 8}, {7, 8}, {3, 9}, {6, 9}, {1, 10}, {8, 10}, {9, 10}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {1, 7}, {2, 8}, {6, 8}, {3, 8}, {3, 6}, {4, 9}, {7, 9}, {3, 9}, {7, 10}, {9, 10}, {6, 10}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {4, 5}, {4, 6}, {5, 6}, {6, 7}, {2, 8}, {6, 8}, {3, 8}, {3, 6}, {4, 9}, {7, 9}, {3, 9}, {6, 9}, {1, 10}, {2, 10}, {7, 10}}],  
 FromUnorderedPairs[{{1, 5}, {1, 6}, {2, 6}, {4, 5}, {4, 6}, {5, 6}, {2, 7}, {2, 8}, {6, 8}, {3, 8}, {3, 6}, {4, 9}, {7, 9}, {3, 9}, {6, 9}, {1, 10}, {6, 10}, {7, 10}}];

$g_{18} = \{ \}$

#### Computations in proving Theorem 4.6.

In the following, let  $grow[X] = grow[X, \{(W_5+e)^*\}]$ .

$g_{11} = grow[W_5] = \{$

FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}}],  
 FromUnorderedPairs[{{2, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 7}, {3, 7}, {6, 7}}];

$g_{12} = grow[g_{11}] = \{$

FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {1, 4}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 4}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}}],

FromUnorderedPairs[{{2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 7}, {6, 7}, {1, 7}}],  
 FromUnorderedPairs[{{2, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {1, 7}, {3, 7}, {6, 7}}],  
 FromUnorderedPairs[{{2, 6}, {3, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {1, 7}, {4, 7}, {6, 7}}],  
 FromUnorderedPairs[{{1, 6}, {3, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 7}, {4, 7}, {6, 7}}],  
 FromUnorderedPairs[{{5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 7}, {3, 7}, {6, 7}, {2, 8}, {4, 8}, {6, 8}}];

$g_{13} = \text{grow}[g_{12} \cup \{W_6\}] = \{$

FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {1, 4}, {2, 4}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {1, 4}, {2, 5}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 7}, {4, 7}, {1, 7}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {1, 4}, {2, 7}, {5, 7}, {1, 7}}],  
 FromUnorderedPairs[{{2, 6}, {3, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 4}, {3, 7}, {6, 7}, {1, 7}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 4}, {1, 7}, {6, 7}, {3, 7}}],  
 FromUnorderedPairs[{{2, 6}, {3, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 4}, {1, 7}, {4, 7}, {6, 7}}],  
 FromUnorderedPairs[{{1, 6}, {3, 6}, {4, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 4}, {2, 7}, {5, 7}, {6, 7}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {3, 7}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {6, 7}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {4, 5}, {2, 7}, {5, 7}, {1, 7}, {1, 8}, {4, 8}, {3, 8}}],  
 FromUnorderedPairs[{{2, 6}, {3, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {1, 8}, {4, 8}, {6, 8}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {4, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {3, 8}, {5, 8}, {6, 8}}],  
 FromUnorderedPairs[{{3, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 7}, {4, 7}, {6, 7}, {2, 8}, {6, 8}, {1, 8}}],  
 FromUnorderedPairs[{{1, 7}, {2, 7}, {3, 7}, {4, 7}, {5, 7}, {6, 7}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {1, 6}, {1, 4}}];

$g_{14} = \text{grow}[g_{13}] = \{$

FromUnorderedPairs[{{2, 6}, {3, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 4}, {4, 7}, {6, 7}, {1, 7}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 7}, {6, 7}, {3, 7}, {1, 8}, {6, 8}, {4, 8}}],  
 FromUnorderedPairs[{{2, 6}, {3, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {3, 7}, {6, 7}, {1, 7}, {1, 8}, {6, 8}, {4, 8}}],  
 FromUnorderedPairs[{{2, 6}, {3, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 4}, {3, 7}, {6, 7}, {1, 7}, {1, 6}}],  
 FromUnorderedPairs[{{2, 6}, {3, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 4}, {3, 7}, {6, 7}, {1, 7}, {1, 3}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {3, 7}, {6, 7}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {6, 8}, {7, 8}, {3, 8}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {4, 5}, {2, 7}, {5, 7}, {1, 7}, {6, 7}, {1, 8}, {4, 8}, {3, 8}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {2, 7}, {5, 7}, {1, 7}, {3, 7}, {1, 8}, {6, 8}, {3, 8}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {1, 4}, {2, 4}, {2, 5}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {1, 4}, {2, 4}, {2, 7}, {5, 7}, {1, 7}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 5}, {2, 7}, {4, 7}, {1, 7}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 7}, {4, 7}, {1, 7}, {6, 7}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {2, 7}, {4, 7}, {1, 7}, {3, 8}, {5, 8}, {1, 8}}],

FromUnorderedPairs[{{1, 6}, {3, 6}, {4, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 7}, {4, 7}, {1, 7}, {2, 8}, {5, 8}, {6, 8}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {4, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 7}, {4, 7}, {1, 7}, {3, 8}, {5, 8}, {6, 8}}],  
 FromUnorderedPairs[{{1, 6}, {3, 6}, {5, 6}, {2, 3}, {4, 5}, {2, 7}, {5, 7}, {1, 7}, {1, 8}, {4, 8}, {3, 8}, {2, 9}, {4, 9}, {6, 9}}];

Let  $G_1, G_2, \dots, G_9$  be the nine graphs in Figure 4.13. Then

$G_1=g_{14}[1], G_2=g_{14}[2], G_3=g_{14}[3], G_4=g_{14}[4], G_5=g_{14}[5], G_6=g_{14}[6], G_7=g_{14}[7], G_8=g_{14}[8], G_9=g_{14}[9]$ .

$h_{14}=g_{14}[\{10, 11, 12, 13, 14, 15, 16, 17\}]$

$h_{15}=\mathbf{grow}[h_{14}]={$

FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {1, 4}, {2, 4}, {2, 5}, {3, 5}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 4}, {2, 4}, {2, 5}, {3, 7}, {5, 7}, {1, 7}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {1, 3}, {1, 4}, {2, 4}, {2, 7}, {5, 7}, {1, 7}, {3, 8}, {5, 8}, {4, 8}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 5}, {2, 7}, {4, 7}, {1, 7}, {6, 7}}],  
 FromUnorderedPairs[{{1, 6}, {2, 6}, {4, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 5}, {2, 7}, {4, 7}, {1, 7}, {3, 8}, {5, 8}, {6, 8}}],  
 FromUnorderedPairs[{{1, 6}, {3, 6}, {4, 6}, {2, 3}, {3, 4}, {4, 5}, {2, 7}, {4, 7}, {1, 7}, {3, 8}, {5, 8}, {1, 8}, {2, 9}, {5, 9}, {6, 9}}],  
 FromUnorderedPairs[{{3, 6}, {5, 6}, {2, 3}, {4, 5}, {2, 7}, {5, 7}, {1, 7}, {1, 8}, {4, 8}, {3, 8}, {2, 9}, {4, 9}, {1,10}, {9,10}, {6,10}}];

$K_6=h_{15}[1], A_1=h_{15}[2], A_2=h_{15}[3], A_3=h_{15}[6], \text{ Petersen}=h_{15}[7]$

$h_{16}=\mathbf{grow}[h_{15}[\{4,5\}]]={$

FromUnorderedPairs[{{3, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 5}, {2, 7}, {4, 7}, {1, 7}, {6, 7}, {1, 8}, {2, 8}, {4, 8}, {6, 8}}];

$\mathbf{grow}[G_1]={$

FromUnorderedPairs[{{2, 6}, {3, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 3}, {2, 4}, {4, 7}, {6, 7}, {1, 7}, {1, 4}}];

$\mathbf{grow}[\{G_2, G_3\}]=$

FromUnorderedPairs[{{1, 6}, {2, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 7}, {6, 7}, {3, 7}, {1, 8}, {6, 8}, {4, 8}, {1, 3}}],  
 FromUnorderedPairs[{{2, 6}, {3, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {3, 7}, {6, 7}, {1, 7}, {1, 8}, {6, 8}, {4, 8}, {1, 3}}],  
 FromUnorderedPairs[{{2, 6}, {3, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {3, 7}, {6, 7}, {1, 7}, {1, 8}, {6, 8}, {4, 8}, {1, 4}}],  
 FromUnorderedPairs[{{2, 6}, {3, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {3, 7}, {6, 7}, {1, 7}, {1, 8}, {6, 8}, {4, 8}, {4, 6}}];

$\mathbf{grow}[\{G_4, G_5\}, \{(W_5+e)^*, G_1, G_3\}]=$

FromUnorderedPairs[{{2, 6}, {3, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 4}, {3, 7}, {6, 7}, {1, 7}, {1, 6}, {1, 3}}],  
 FromUnorderedPairs[{{2, 6}, {4, 6}, {5, 6}, {1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}, {1, 4}, {3, 7}, {6, 7}, {1, 7}, {1, 8}, {3, 8}, {6, 8}}];

$p_{14}=\{G_6, G_8, G_9\};$

$p_{15} = \text{grow}[p_{14}, \{(W_5+e)^*, G_5, G_7\}] = \{$

FromUnorderedPairs[{{1, 6}, {2, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {2, 7}, {5, 7}, {1, 7}, {3, 7}, {6, 7}, {1, 8}, {6, 8}, {3, 8}}],  
FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {4, 5}, {2, 7}, {5, 7}, {1, 7}, {6, 7}, {1, 8}, {4, 8}, {3, 8}, {6, 8}}],  
FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {4, 5}, {2, 7}, {5, 7}, {1, 8}, {4, 8}, {3, 8}, {1, 9}, {6, 9}, {7, 9}}];

$p_{16} = \text{grow}[p_{15}, \{(W_5+e)^*, G_5, G_7\}] = \{$

FromUnorderedPairs[{{1, 6}, {2, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {2, 7}, {5, 7}, {1, 7}, {3, 7}, {6, 7}, {1, 8}, {6, 8}, {3, 8}, {3, 6}}],  
FromUnorderedPairs[{{1, 6}, {2, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {5, 7}, {1, 7}, {3, 7}, {1, 8}, {6, 8}, {3, 8}, {2, 9}, {6, 9}, {7, 9}}],  
FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {4, 5}, {2, 7}, {5, 7}, {1, 8}, {4, 8}, {3, 8}, {6, 8}, {1, 9}, {6, 9}, {7, 9}}];

$p_{17} = \text{grow}[p_{16}, \{(W_5+e)^*, G_5, G_7\}] = \{$

FromUnorderedPairs[{{1, 6}, {2, 6}, {4, 6}, {5, 6}, {3, 4}, {4, 5}, {2, 7}, {5, 7}, {1, 7}, {3, 7}, {6, 7}, {1, 8}, {6, 8}, {3, 8}, {2, 9}, {6, 9}, {3, 9}}],  
FromUnorderedPairs[{{1, 6}, {2, 6}, {3, 6}, {4, 6}, {5, 6}, {2, 3}, {4, 5}, {2, 7}, {5, 7}, {1, 8}, {4, 8}, {3, 8}, {6, 8}, {1, 9}, {6, 9}, {7, 9}, {6, 7}}];

$p_{18} = \text{grow}[p_{17}, \{(W_5+e)^*, G_5, G_7\}] = \{$

FromUnorderedPairs[{{1, 6}, {2, 6}, {4, 6}, {5, 6}, {3, 4}, {4, 5}, {2, 7}, {5, 7}, {1, 7}, {3, 7}, {6, 7}, {1, 8}, {6, 8}, {3, 8}, {2, 9}, {6, 9}, {3, 9}, {3, 6}}];

$p_{19} = \text{grow}[p_{18}, \{(W_5+e)^*, G_5, G_7\}] = \{;$

$\text{grow}[\{G_7\}, \{(W_5+e)^*, G_5\}] = \{$

FromUnorderedPairs[{{1, 6}, {2, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {6, 8}, {7, 8}, {3, 8}, {3, 6}}],  
FromUnorderedPairs[{{1, 6}, {2, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {6, 8}, {7, 8}, {3, 8}, {3, 7}}];

Eleven ways of splitting vertex  $z$  in  $G_7 + zx_1 = \{$

FromUnorderedPairs[{{4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {6, 8}, {7, 8}, {3, 8}, {6, 7}, {1, 9}, {2, 9}, {6, 9}}],  
FromUnorderedPairs[{{2, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {6, 8}, {7, 8}, {3, 8}, {6, 7}, {1, 9}, {4, 9}, {6, 9}}],  
FromUnorderedPairs[{{2, 6}, {4, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {6, 8}, {7, 8}, {3, 8}, {6, 7}, {1, 9}, {5, 9}, {6, 9}}],  
FromUnorderedPairs[{{2, 6}, {4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {6, 8}, {7, 8}, {3, 8}, {1, 9}, {7, 9}, {6, 9}}],  
FromUnorderedPairs[{{1, 6}, {2, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {6, 8}, {7, 8}, {3, 8}, {6, 7}, {4, 9}, {5, 9}, {6, 9}}],  
FromUnorderedPairs[{{1, 6}, {2, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {6, 8}, {7, 8}, {3, 8}, {4, 9}, {7, 9}, {6, 9}}],  
FromUnorderedPairs[{{1, 6}, {2, 6}, {4, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {6, 8}, {7, 8}, {3, 8}, {5, 9}, {7, 9}, {6, 9}}],  
FromUnorderedPairs[{{5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {6, 8}, {7, 8}, {3, 8}, {6, 7}, {1, 9}, {2, 9}, {4, 9}, {6, 9}}],  
FromUnorderedPairs[{{4, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {6, 8}, {7, 8}, {3, 8}, {6, 7}, {1, 9}, {2, 9}, {5, 9}, {6, 9}}],  
FromUnorderedPairs[{{4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {6, 8}, {7, 8}, {3, 8}, {1, 9}, {2, 9}, {7, 9}, {6, 9}}],  
FromUnorderedPairs[{{4, 6}, {5, 6}, {2, 3}, {3, 4}, {4, 5}, {1, 3}, {2, 7}, {5, 7}, {1, 7}, {7, 8}, {3, 8}, {6, 7}, {1, 9}, {2, 9}, {8, 9}, {6, 9}}];