Print Your Name Here: ____________________

- **Show all work** in the space provided. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side.*
- **No** books or notes (paper or electronic) or communication devices (smart/cell phones, internet-connected devices such as laptops, tablets, or I-watches) are allowed. A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. ***Do not*** replace precise answers such as $\sqrt{2}$, $\frac{1}{3}$, or π with decimal approximations. *Keep your eyes on your own paper!*
- The maximum score possible is 100 points.

1. (20 points) Find $\int_C (x - y^2)\,dx + y\,dy$ if C is the straight line segment from (1,0) to (3,1).

2. (20 points) Let $\mathbf{F}(x, y) = \langle 3x^2 + y^2, 2xy + 1\rangle$ and let C be the lower semicircle $y = -\sqrt{4 - x^2}$, $-2 \leqslant x \leqslant 2$.

a. (5) Show *using appropriate partial derivatives* that $\mathbf{F}$ is conservative.

b. (10) Find a *potential function* f for $\mathbf{F}$. (That is, we need $\mathbf{F} = \nabla f$.)

c. (5) Use the Fundamental Theorem for Line Integrals to find $\int_{(-2,0)}^{(2,0)} (3x^2 + y^2)\,dx + (2xy + 1)\,dy$.

3. (20 points) *Use Green's Theorem* to evaluate $\oint_C \left(3+e^{x^2}\right)\,dx + (Tan^{-1}y + 3x^2)\,dy$ where C is the positively oriented curve enclosing the region R that is bounded by the *first quadrant* part of the circle $x^2+y^2=1$ and the two axes.

4. (20 points) Let $\mathbf{F}(x,y,z) = \langle 2xy^3z^2,\, 3x^2y^2z^2,\, 2x^2y^3z\rangle$.

a. (10) Find $\nabla \times \mathbf{F}$, the *curl* of $\mathbf{F}$.

b. (10) Is $\mathbf{F}$ conservative? If yes, then find a *potential function* f for $\mathbf{F}$.

5. (20 points) *Use Stokes' Theorem* to evaluate $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$ where S is the *upward oriented upper* hemisphere $z = \sqrt{1 - x^2 - y^2}$ with the boundary curve C being $x^2 + y^2 = 1$ in the xy-plane. Here $\mathbf{F}(x, y, z) = \langle ze^y,\, x\cos z,\, xz\sin y\rangle$.

Solutions

1. This is NOT a path-independent integral! And C is not a closed curve: it does not enclose a region, so Green's theorem is irrelevant. For this path, $x = 1 + 2t$ and $y = t$, $0 \leqslant t \leqslant 1$. Thus $\int_C (x - y^2)\,dx + y\,dy = \int_0^1 (1 + 2t - t^2)2\,dt + t\,dt = 2 + \frac{5}{2} - \frac{2}{3} = \frac{23}{6}$. Compare with 16.2/43.

2. Compare with 16.3/13.

a. (5) Letting M and N be the **i** and **j** components of **F** respectively, we have $M_y = 2y = N_x$.

b. (10) $f(x, y) = x^3 + xy^2 + y$. Many students were very awkward finding f. Please learn the method I taught you, which is straight forward and systematic. One could add an arbitrary constant, but this is not necessary since any potential function for **F** will suffice.

c. (5) $\int_{(-2,0)}^{(2,0)} (3x^2 + y^2)\,dx + (2xy + 1)\,dy = f(2, 0) - f(-2, 0) = 16$.

3. $\oint_C \left(3 + e^{x^2}\right)\,dx + (Tan^{-1}y + 3x^2)\,dy = \iint_R 6x\,dA = \int_o^{\frac{\pi}{2}} \int_0^1 (6r\cos\theta)r\,dr\,d\theta = 2$. Many students forgot how to integrate over a plane region in polar coordinates. Also, one must be able to distinguish one quarter of a unit *disc* from a unit *square*! And of course one must know Green's Theorem. Compare with 16.4/13.

4. Compare with 16.5/15.

a. (10) $\nabla \times \mathbf{F} = \langle 0, 0, 0 \rangle = \mathbf{0}$. Note that the curl of a vector field is always a vector.

b. (10) **F** is conservative and $f(x, y, z) = x^2y^3z^2$. Again, one may add an arbitrary constant, but this is not necessary since any potential function will suffice.

5. $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\,dS = \oint_C ze^y\,dx + x\cos z\,dy + xz\sin y\,dz = \oint_C x\cos 0\,dy = \oint_C x\,dy = \iint_{x^2+y^2\leqslant 1} 1\,dA = \pi$. Here we used the fact that $z = 0$ in the xy-plane and we used Green's theorem to evaluate the last of the line integrals. But you could evaluate the line integral directly. Compare with 16.8/3.

Class Statistics

% Grade	Test#1	Test#2	Test#3	Final Exam	Final Grade
90-100 (A)	13	16	14		
80-89 (B)	6	7	6		
70-79 (C)	6	4	5		
60-69 (D)	5	1	1		
0-59 (F)	0	2	4		
This Test Avg	84.8%	86.0%	81.6%	%	%
HW Avg	87.5%	88.1%	88.4%	%	%
HW/TST Corr	0.61	0.77	0.70		

The Correlation Coefficient is the cosine of the angle between two data vectors in $\mathbb{R}^{31}$–one dimension for each student enrolled. Thus this coefficient is between 1 and -1, with coefficients above 0.6 being considered strongly positive. The correlation coefficient shown indicates that the test grades in the course have a strongly positive correlation with performance on the homework.