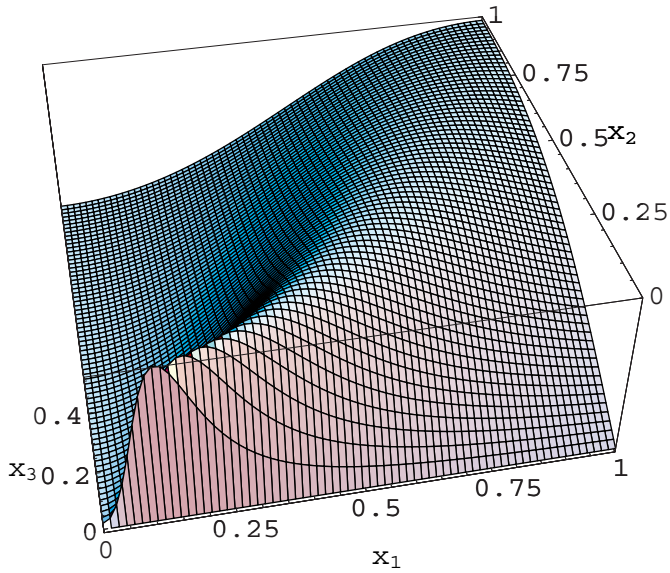
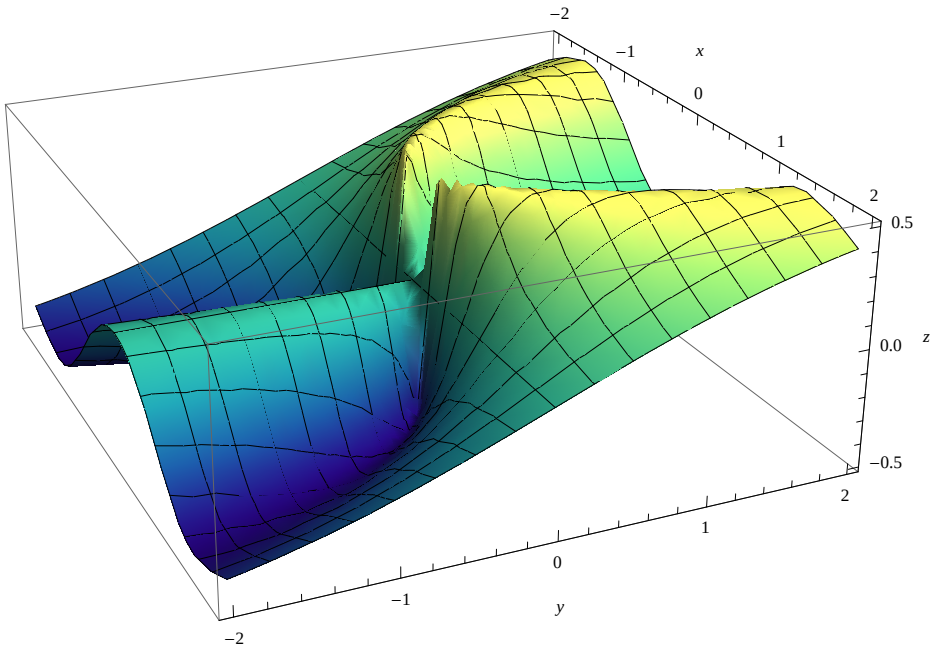


Here $f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$. Show $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist. What happens as $(x,y) \to (0,0)$ along:

(a) the y-axis? (Find $\lim_{y\to 0} f(0,y)$.)

(b) any line $y = kx$?

(c) the parabola $y = x^2$?



Now look at a second picture showing a larger part of the same surface - this time with the origin at the center of the box shown. Do you see what part of the second picture is shown from a different angle in the first picture?