What do I learn if I decide to compute integrals?

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The study of the evaluation of definite integrals is full of surprises.

In this talk will present some of them in relation to Dynamical Systems, Number Theory and Combinatorics.

The first example deals with the rational integral

$$N_{0,4}(a;m) := \int_0^\infty \frac{dx}{(x^4 + 2ax^2 + 1)^{m+1}}.$$

The numbers

$$d_l(m) := 2^{-2m} \sum_{k=l}^m 2^k \binom{2m - 2k}{m - k}\binom{m + k}{k}\binom{k}{l}$$

play an important role in its evaluation.

The sequence $\{d_l(m) : 0 \le l \le m\}$ has many intriguing properties, some of which remain to be decided. (= *Looking for collaborators*).

Here is a nice non-linear dynamical system

$$a \quad \mapsto \quad \frac{ab + 5a + 5b + 9}{(a + b + 2)^{4/3}}$$

$$b \quad \mapsto \quad \frac{a + b + 6}{(a + b + 2)^{2/3}}$$

Where do these formulas come from?

Integrals, of course.