Targeting Misconceptions and Improving Teaching Methods for Vector Calculus

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Introduction

Vector Calculus is a field of mathematics that a diverse population of students may encounter in college.

This poster concentrates on:

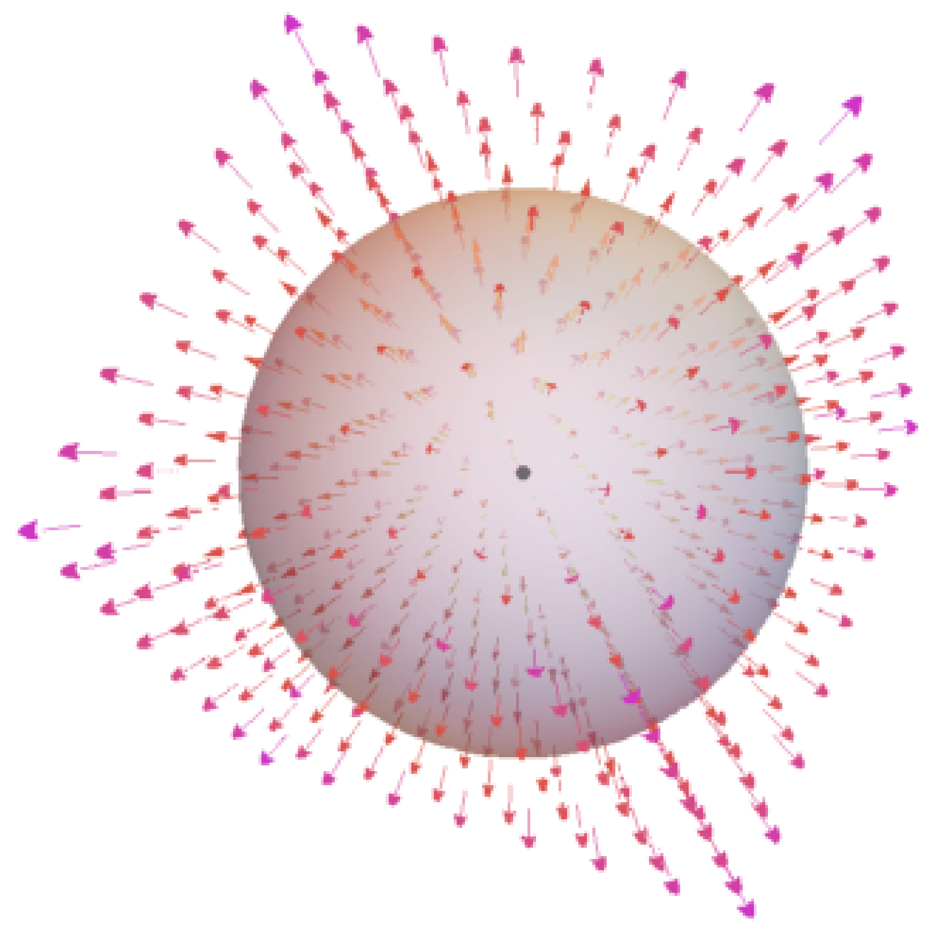
- Teaching the divergence theorem
- Analyzing learning styles of four students with differing majors (Math, Physics, Engineering, Non-Science)
- Investigating the acquired misconceptions of these students
- Addressing misconceptions one-on-one and in a classroom setting
- Employing and proposing specific pedagogical methods for instructors of Vector Calculus

I discovered that an instructor's preparation can influence:

- The success or failure of an attempt to appeal to student aims
- The quality and efficiency of explanations and student resources
- The classroom learning environment

The Divergence Theorem

This figure depicts a closed three-dimensional region in the presence of a vector field.



- Let $\mathbf{F}$ be a vector field in space and G be a three-dimensional region.
- This three-dimensional region has a surface area ∂G that serves as a boundary for the region.
- At every point in this space $\mathbf{F}$ prescribes a vector; whether this is within the region, outside of the region, or on the boundary of the region.
- In order to calculate the flux of $\mathbf{F}$ through ∂G, we must look at the magnitude of the components of the vectors in $\mathbf{F}$ that are pointing in the normal direction of the surface.
- According to the Divergence Theorem, the flux of $\mathbf{F}$ through ∂G is equal to the sum of the flux through every small unit of volume contained with in the region, and, most importantly, that these fluxes are obtained by a flux density, called the divergence $\nabla \cdot \mathbf{F}$.

The theorem is as follows:

$$\int_{\partial G} \mathbf{F} \cdot \mathbf{n}\, dG = \int_{G} \nabla \cdot \mathbf{F}\, dV$$

So, by the Divergence Theorem if we calculate the net flux through all of the tiny pieces of volume by integrating the divergence, this calculation will be equal to the flux of the vector field through the boundary of this three-dimensional region.

Comparing Student Learning

I collected data to mark the students' understanding of the Divergence Theorem before and after the group lesson. *(Initial $\longrightarrow$ Final Comprehension)*

STUDENT	LEVEL OF COMPREHENSION		
	CONCEPTUAL	MATHEMATICAL	APPLICATION
MATH	✓- $\longrightarrow$ ✓++	✓- $\longrightarrow$ ✓+	✓- $\longrightarrow$ ✓++
ENGINEERING	✓ $\longrightarrow$ ✓++	✓ $\longrightarrow$ ✓+	✓ $\longrightarrow$ ✓++
PHYSICS	✓- $\longrightarrow$ ✓++	✓- $\longrightarrow$ ✓	✓- $\longrightarrow$ ✓+
NON-SCIENCE	✓- $\longrightarrow$ ✓+	✓- $\longrightarrow$ ✓	✓- $\longrightarrow$ ✓

What Benefited Students:

1. Conceptual Explanations
   - 4 /4 students
2. Application
   - 3/4 students *(Exception: Math)*
3. Explanation of Notation
   - 2/4 students *(Exception: Math, Engineering)*
4. Hand-Drawn Visual Aid
   - 2/4 students *(Exception: Engineering, Physics)*
5. Computer Graphic Visual Aid
   - 2/4 Students *(Exception: Engineering, Physics)*
6. Real World Problems
   - 1/4 Students *(Exception: Math, Physics, Non-Science)*

Student Learning Obstacles

Misconceptions and Suggested Solutions

- **The vector field $\mathbf{F}$ moves the three-dimensional figure.**
  Emphasize that for this problem the three-dimensional region is not physical. Explain precisely what the problem is and what the problem is not.
- **The Divergence Theorem states that the rate at which the fluid or particles are moving into the region is equal to the rate at which they are moving out of the region.**
  Explain that this is only true in a specific case of the Divergence Theorem where the divergence is 0. Then, carefully explain that the theorem associates two different ways to compute the flux, or flow, of a physical quantity out of a region in space.
- **In the theorem statement, the $\nabla \cdot \mathbf{F}$ means you take the gradient of $\mathbf{F}$.**
  Carefully label different sections of the mathematical statement of the theorem and explain the meaning of the notation used. Explain that $\nabla\cdot$ is the "divergence operator" will result in a scalar function, since $\nabla \cdot \mathbf{F}$ is a dot product, whereas taking the gradient of $\mathbf{F}$ will result in a vector function.

Complications

- **Lack of specified units makes problems less accessible.**
  Prepare problems with specified units for the students in advance, in order to show how the units work out in the theorem.
- **Passage between examples from a general figure to a specific figure.**
  Briefly draw several possible figures first before proceeding to draw a general figure that may represent any of the several and more, to emphasize the flexibility of the theorem.
- **Intuitive explanation is necessary but not adequate for deeper understanding.**
  To conserve time and hold the students' attention, practice a brief, precise explanation for interested students. Explain why the divergence of $\mathbf{F}$ times the volume of a small cube is equal to the flux of $\mathbf{F}$ out of this cube.

Classroom Challenges

Difficulties

1. Using specific references to appeal to all students in a class without over-complicating the concepts or taking too much time.
2. Students have differing experiences with notation, some students may need more explanation than other students.
3. Students may have varying needs when it comes to visual aid. Some students may not be able to reproduce the drawings necessary to understand various problems.

Recommended Solutions

1. Quickly mention several examples that will resonate with various students in order to showcase the broad application of the concept.
2. Regardless of whether or not students have seen certain notation, carefully explain the meaning of each part. This will help reinforce the concepts to the experienced students and simultaneously facilitate the learning of the inexperienced students.
3. Supply the students with plenty of resources to explore for visual aid, such as websites or programs that can generate the visuals they may have difficulty producing on their own. Practice drawing visuals you intend to draw and relate to explanations or problems in class.

Personal Challenges

1. Transitioning from one concept to another
   - Improved by talking through my explanations with others to assess my thought process.
2. Making assumptions about what the students knew
   - Attempted to fix this by carefully interviewing students to assess their knowledge.
3. Self-Confidence in teaching students as a student
   - Improved by spending much time with the material and frequently talking through the material with multiple people.
4. Appealing to all student aims
   - Improved by interviewing students to see what types explanations they preferred. (i.e. real world application problems, intuitive physical explanations, or a straightforward mathematical statements)

Conclusions

Instructor Preparation Influences:

1. The success or failure of an attempt to appeal to student aims
   - An instructor should be cognizant of the major classification demographic of their class as much as possible in order to foster to the needs of the class.
2. The quality and efficiency of explanations and student resources
   - An instructor who prepares a practiced and clear explanation, has the ability to create a connection between the students and the concepts in question.
3. The classroom learning environment
   - An instructor who creates an open-learning environment will allow the students to achieve deeper understanding of concepts by learning from their misconceptions or mistakes.

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