Spring Mini Courses in Analysis and Geometry

Abstracts

February 8 - 11, 2018

Feichtinger conjecture, *Peter G. Casazza*

(1) We start with a brief introduction to frame theory. Next we look at the origins of the Feichtinger Conjecture in Signal Processing. We will then look at the most recent solutions to the Feichtinger Conjecture and the best constants obtained so far.
(2) We will look at equivalent forms of the Feichtinger Conjecture including the Kadison-Singer Problem, The Paving Conjecture, The Projection Paving Conjecture, the R(epsilon) Conjecture, the Bourgain-Tzifriri Conjecture, the Harmonic Analysis Conjecture,
(3) The solution to the Feichtinger Conjecture due to Marcus, Spielman, and Srivastave using the Weaver Conjecture. The Casazza/Tremain Conjecture and its strongest form available today.

Fuglede conjecture, *Palle E.T. Jorgensen*

(1) Vector fields and spectral theory: History of the problem: paper by Fuglede, connections to I.E. Segal and to J. von Neumann. Unbounded Hermitian operators, selfadjoint extensions, boundary values and spectral theory. Some non-commutative analogues.
(2) The two sides of the Fuglede-conjecture/problem: A closer look at tiles vs orthogonal Fourier frequencies (spectral pairs). Tao et al, dimensions 3 and higher, vs dimension 1 and 2. The case of d-cubes. The universal tiling conjecture.
(3) Fractals and representation theory: Fractal variants of the conjecture. Scaling, self-similarity, fractal limits, wavelets on fractals, Fourier series on affine fractals, spectral pairs revisited, fractals in the large. Ergodic theoretic constructions. Representations of the Cuntz relations.

Roza Aceska, Dynamical sampling and frames generated by iterative actions of operators

Abstract: Dynamical sampling refers to the process of sampling an evolving signal f at various time instances, where the evolutionary changes of f are described as an output under the iterative actions of some operator. It is known that the problem of signal recovery from the coarse samples taken over time is equivalent to ensuring that the system generated by iterative actions of operators satisfies the frame inequality. We review some of the recent developments and properties of frames generated by iterations, such as scalability and stability.

Jens Christensen, Atomic decompositions of Bergman spaces on tube type domains

Abstract: We derive atomic decompositions for Bergman spaces on tube type domains by taking Laplace extensions of atoms on Besov spaces on the cone. We then discuss how this extends classical results due to Coifman and Rochberg and relate it to a concurrent result by Bekolle, Gonessa, and Nana.

Emily King, Fourier Analysis on Groups and Grassmannian Packings

Abstract: It is often of interest to find subspaces which yield a resolution of the identity and are also optimally spread apart. Such objects are called Grassmannian (fusion) frames. There are a number of constructions possible using tools from combinatorial design theory and algebraic combinatorics. In this talk, a number of related constructions will be presented which arise from difference sets and their generalizations. The reasons the constructions work is due to Fourier analysis on finite groups.

Azita Mayeli, Smooth and symmetric convex sets with no orthogonal Gabor bases

Abstract: Let K be a convex and symmetric bounded set in $\mathbb{R}^d$, $d \geq 2$, with smooth boundary. Using a combinatorial approach, in this talk we show that for $d \neq 1(mod4)$, the indicator function of K can not serve as an orthogonal Gabor window function for $L^2(\mathbb{R}^d)$, i.e., there is no countable set $S \subset \mathbb{R}^{2d}$ such that the Gabor family $\mathcal{G}(1_K, S) = \{e^{2\pi i x \cdot b} 1_K(x-a) : (a,b) \in S\}$ is an orthogonal basis for $L^2(\mathbb{R}^d)$.

Eyvindur Palsson, Falconer type theorems for higher order point configurations

Abstract: Finding and understanding patterns in data sets is of significant importance in many applications. One example of a simple pattern is the distance between data points, which can be thought of as a 2-point configuration. Two classic questions, the Erdos distinct distance problem, which asks about the least number of distinct distances determined by N points in the plane, and its continuous analog, the Falconer distance problem, explore that simple pattern. Questions similar to the Erdos distinct distance problem and the Falconer distance problem can also be posed for more complicated patterns such as patterns based off of three points, which can be viewed as 3-point configurations. In this talk I will briefly explore such generalizations. The main techniques used come from analysis and geometric measure theory.

Hans D. Parshall, Falconer-type problems over finite fields

Abstract: The Iosevich-Rudnev distance theorem tells us that large subsets of vector spaces over finite fields determine all possible distances. We will discuss recent progress in this setting on locating isometric copies of point configurations and highlight connections with the Euclidean setting.

Boris Rubin, Fuglede's Formula for Horospherical Transforms.

Susanna Spektor, The approximation of almost-time and band-limited functions by their expansion in some orthogonal polynomial base

Abstract: In this joint work with Philippe Jaming and Abderrazek Karoui our aim is to investigate the quality of approximation of almost time- and almost band-limited functions by its expansion in two classical orthogonal polynomials bases: the Hermite basis and the ultraspherical polynomials bases (which include Legendre and Chebyshev bases as particular cases). This allows us to obtain the quality of approximation in the L^2-Sobolev space by these orthogonal polynomials bases. Also, we obtain the rate of the Legendre series expansion of the prolate spheroidal wave functions.