# SYLLABUS FOR THE FIRST YEAR GRADUATE ALGEBRA SEQUENCE (MATH 7210–7211)

(Revised May 2007)

1. **Groups**
   (a) Basic notions
   (b) The isomorphism theorems
   (c) Symmetric and alternating groups, Cayley's theorem
   (d) Group actions and the class equation
   (e) Sylow theorems
   (f) Fundamental theorem on finitely generated abelian groups (proof postponed until theorem on modules over a PID)
   (g) Normal series and solvable groups, Jordan-Hölder theorem

2. **Rings and Modules**
   (a) Basic notions and constructions
   (b) Polynomial rings
   (c) Field of fractions
   (d) Euclidean domains, PID's, UFD's
   (e) Structure theorem for finitely generated modules over a PID
   (f) Applications to abelian groups and Jordan canonical form
   (g) Exact sequences
   (h) Tensor products and Hom
   (i) Noetherian (commutative) rings
   (j) Hilbert Basis Theorem

3. **Field Theory**
   (a) Field extensions
   (b) Finite fields
   (c) Normality and separability
   (d) The fundamental theorem of Galois theory
   (e) Applications (eg insolvability of the quintic, geometric constructions)

4. **Algebras over a field**
   (a) Definitions and examples–matrix algebras, quaternions, group algebras
   (b) Jacobson radical
   (c) Simple and semisimple algebras–Wedderburn theorem
   (d) Group representations and Maschke's theorem

(e) Multilinear algebra

The algebra sequence will also expose students to the basic language of category theory.

Here is the breakdown of topics into the two semester, together with a suggested approximate ordering of the topics within each semester:

**Math 7210:** 1a-f, 2a-f, 3a-b.
**Math 7211:** 3c-e, 1g, 2g-j, 4a-e.