

# Abstract Volterra Equations

by Mihi Kim, May 1995

## ABSTRACT

This dissertation is devoted to the study of the abstract Volterra equation

$$v(t) = A \int_0^t v(t-s) d\mu(s) + f(t) \quad \text{for } t \geq 0 \quad (\text{VE})$$

where  $A$  is a closed linear operator in a complex Banach space  $X$ ,  $\mu$  is a complex valued function of bounded variation, and  $f : [0, \infty) \rightarrow X$  is continuous and Laplace transformable. Laplace transform methods are used to characterize the existence and uniqueness of exponentially bounded solutions  $v$  for a given forcing term  $f$ , an operator  $A$ , and a given kernel  $\mu$ . We extend the methods of a solution family (or a resolvent) for (VE) by studying integrated and analytic integrated solution operator families. These notions are employed to characterize those pairs  $(A, \mu)$  for which (VE) has unique solutions for all sufficiently regular forcing terms  $f$ . Besides existence, uniqueness and wellposedness results for (VE), new results include Trotter-Kato type theorems for integrated solution operator families and a characterization of those pairs  $(A, \mu)$  for which the integrated solution operator families are analytic in an open sector  $\{\lambda \in \mathbb{C} \mid |\arg \lambda| < \alpha\}$  for some  $\alpha \in (0, \frac{\pi}{2}]$ .