Core II Exam
Graph Theory
August 2001

Solve any six from among the following eight problems. Submit only the six selected problems. You have 3 and 1/2 hours. Good Luck!

Graphs are undirected, and may have loops and multiple edges. Simple graphs are those with no loops and no multiple edges.

1. Use Euler's Formula to determine the largest possible number of egdes in a simple outerplanar graph on n vertices.

2. Determine the chromatic number of the Petersen graph.

3. (a) State Kuratowski's Theorem.
(b) Use Kuratowski's Theorem to show that a graph is outerplanar if and only if none of its subgraphs is isomorphic to a subdivision of K_4 or $K_{2,3}$.

4. Let G be a connected plane graph and let G^* be a plane dual of G. Complete and prove the following:
G is Hamiltonian if and only if the vertex set of G^* can be partitioned into two sets such that the subgraph of G^* induced by each set is ...

5. Suppose G is a 3-regular graph with a cut-edge. What is the edge-chromatic number of G?

6. For a graph G, let $o(G)$ denote the number of connected components of G that have an odd number of vertices. Prove that a tree T has a perfect matching if and only if $o(T - v) = 1$ for every $v \in V(T)$.

7. Prove that a graph with every vertex degree even has no cut-edge.

8. (a) State the König-Egerváry Theorem.
(b) Prove the theorem from part (a) by using Menger's Theorem.