Instructions: *Work two problems from Section A, two problems from Section B, and one problem from Section C, for a total of five problems.* Be sure to write the number of each problem you work out, and write your name clearly at the top of each page you turn in for grading. You have three hours. Good luck!

A. **Point Set Topology** (2 problems)

A1. Show that X is Hausdorff if and only if the diagonal $\Delta = \{x \times x \mid x \in X\}$ is closed in $X \times X$.

A2. i. Show that if $f : X \to Y$ is a continuous bijection from a compact space X to a Hausdorff space Y, then f is a homeomorphism.

ii. Give an example of topological spaces X and Y and a continuous bijection $f : X \to Y$ that is **not** a homeomorphism.

A3. Let $f : X \to Y$ be a quotient map of topological spaces such that Y is connected and for each $y \in Y$, the set $f^{-1}(y)$ is a connected subspace of X. Show that X is connected.

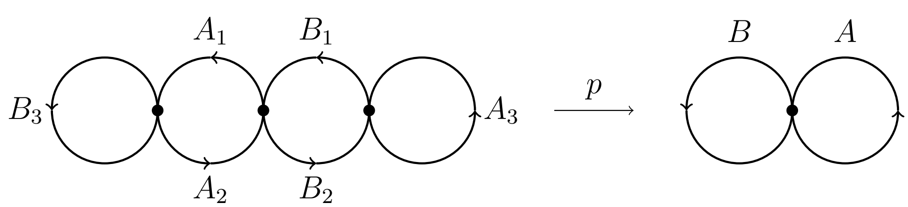
B. **Homotopy** (2 problems)

B1. i. Show that a retract of a contractible space is contractible.

ii. Show that S^1 is not a retract of D^2.

B2. State precisely the Seifert-van Kampen Theorem, and use it to compute the fundamental group of the real projective plane $\mathbb{R}P^2$.

B3. Let $p : E \to X$ be the covering map indicated in the figure below. Determine the group of covering transformations. Is this covering regular?



C. **Mixed** (1 problem)

C1. Let $f : S^1 \to \mathbb{R}$ be a continuous map.

i. Show that there exists a point z of S^1 such that $f(z) = f(-z)$.

ii. Show that f is not surjective.

C2. i. Show that every continuous map $f : \mathbb{R}P^2 \to S^1$ is nullhomotopic.

ii. Find a continuous map $f : S^1 \times S^1 \to S^1$ that is not nullhomotopic.
