Instructions: *Work two problems from Section A, two problems from Section B, and one problem from Section C, for a total of five problems.* Be sure to write the number of each problem you work out, and write your name clearly at the top of each page you turn in for grading. You have three hours. Good luck!

A. **Point Set Topology** (2 problems)

A1. Let $f : X \to Y$ be a function between topological spaces. The *graph* of f is defined by

$$G_f = \{(x, y) \in X \times Y \mid y = f(x)\}.$$

i. Show that if Y is Hausdorff and f is continuous, then G_f is closed.

ii. Show that if Y is compact and G_f is closed, then f is continuous. *(You may use the fact that $\pi_1 : X \times Y \to X$ is a closed map when Y is compact).*

A2. i. Prove that a path-connected space is connected.

ii. Prove that a connected, locally path-connected space is path-connected.

A3. Let $f : X \to Y$ be a quotient map of topological spaces such that Y is connected and for each $y \in Y$, the set $f^{-1}(y)$ is a connected subspace of X. Show that X is connected.

B. **Homotopy** (2 problems)

B1. i. Show that a retract of a contractible space is contractible.

ii. Show that S^1 is not a retract of D^2.

B2. State precisely the Seifert-van Kampen Theorem. Use this theorem to show that the n-sphere S^n is simply connected for $n \geq 2$.

B3. Let B be a path-connected space and $p : E \to B$ a covering map. Let $b_0 \in B$ and assume that $p^{-1}(b_0)$ has k elements.

i. Show that $p^{-1}(b)$ has k elements for every $b \in B$.

ii. Show that if $k = 1$ then p is a homeomorphism.

C. **Mixed** (1 problem)

C1. Let A and B be disjoint compact subspaces of the Hausdorff space X. Show that there exist disjoint open sets U and V containing A and B, respectively.

C2. Let X be path-connected and locally path-connected, and suppose that the fundamental group of X is finite. Prove that any continuous function $f : X \to S^1$ is null-homotopic.
