Instructions: *Do* **any four** *of the following six problems. Be sure to write the number for each problem you work out and write your name clearly at the top of each page you turn in for grading. You have* $2\frac{1}{2}$ *hours.* Good luck!

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1. Prove that for $A \subset X$, $\overline{A} \setminus \operatorname{Int} A = \overline{A} \cap \overline{X \setminus A}$ by showing each set is contained in the other.

2. Let X be a metric space. Prove the following are equivalent.
   (a) X is separable.
   (b) X is second countable.
   (c) X is Lindelöf.

3. Let X be a Hausdorff space with the property that for every $x \in X$, there exists a compact set K (depending on x) such that $x \in \operatorname{Int} K$, the interior of K. Show that the space X is regular.

4. Let $f : X \to Y$ be a continuous function from a compact metric space (X, d) to a metric space (Y, ρ). Show that f is uniformly continuous.

5. Let (X, d) be a metric space.
   (a) Show that a sequence $\{x_n\}$ can converge to at most one point.
   (b) A point $y \in X$ is a *cluster point* of $\{x_n\}$ if given $\epsilon > 0$ and $n \in \mathbb{N}$, there exists $m \geq n$ such that $d(x_m, y) < \epsilon$. Show that the set S of cluster points of $\{x_n\}$ is given by
   $$S = \bigcap_n \overline{\{x_i \mid i \geq n\}}.$$
   (c) Show that if a Cauchy sequence clusters to a point p, then it converges to p.
   (d) Prove or disprove: if a sequence has exactly one cluster point, then it must converge to that point.

6. A map $f : X \to Y$ is *monotone* if $f^{-1}(\{y\})$ is connected for every $y \in Y$. Let $f : X \to Y$ be a surjective monotone quotient mapping. Show that if Y is connected, then X is connected.