Directions: Do exactly three of the following five problems. Do not hand in more than three problems. Please write your name clearly at the top of each sheet turned in. You have 2 and 1/2 hours. Good luck!

1. Give an example of a non-metrizable topological space. Prove your assertion.

2. Prove that a finite Hausdorff space is discrete.

3. Prove that a compact subset of a Hausdorff space is closed.

4. Prove that a closed subset of a compact space is compact.

5. Let $S$ denote the following union of three point-sets in the Euclidean plane $\mathbb{R}^2$:

   \[ S = \{(t, 0) : 0 < t \leq 1\} \cup \{(\frac{1}{n}, s) : n = 1, 2, 3, \ldots \text{ and } 0 \leq s \leq 1\} \cup \{(0, \frac{1}{2})\}. \]

State whether $S$ is connected or not, and prove your assertion.