

CORE I – TOPOLOGY

January, 2003

Directions: Do exactly three of the following five problems. Do not hand in more than three problems. Please write your name clearly at the top of each sheet turned in. You have 2 and 1/2 hours. Good luck!

1. Give an example of a non-metrizable topological space. Prove your assertion.
2. Prove that a finite Hausdorff space is discrete.
3. Prove that a compact subset of a Hausdorff space is closed.
4. Prove that a closed subset of a compact space is compact.
5. Let S denote the following union of three point-sets in the Euclidean plane \mathbf{R}^2 :

$$S = \{(t, 0) : 0 < t \leq 1\} \cup \left\{ \left(\frac{1}{n}, s \right) : n = 1, 2, 3, \dots \text{ and } 0 \leq s \leq 1 \right\} \cup \left\{ \left(0, \frac{1}{2} \right) \right\}.$$

State whether S is connected or not, and prove your assertion.