Instructions: *Work two problems from Section A, two problems from Section B, and one problem from Section C, for a total of five problems.* Be sure to write the number for each problem you work out, and write your name clearly at the top of each page you turn in for grading. You have three hours. Good luck!

A. **Point Set Topology** (2 problems)

A1. Let $f : S^1 \to \mathbb{R}$ be continuous, where S^1 is the unit circle in $\mathbb{R}^2$.

i. Show that there is a point $x \in S^1$ such that $f(x) = f(-x)$

ii. Show that f is not surjective.

A2. i. Let X be a Hausdorff space and A be a compact subset of X. Prove that A is closed.

ii. Let $f : X \longrightarrow Y$ be a continuous map from a compact space X to a Hausdorff space Y. Prove that f is a closed map.

A3. Prove: If A is a connected subspace of a topological space X then the closure of A is connected. Give an example where A is connected, but the interior of A is not connected.

B. **Homotopy** (2 problems)

B1. Let X be a closed surface of genus 2.

i. Using Seifert-van Kampen's theorem, compute the fundamental group of X.

ii. Assuming the classification of finitely generated abelian groups, prove that X is not homeomorphic to a torus.

B2. Let X be the subspace of $\mathbb{R}^2$ that is the union of two circles of radius 1 centered at $(-2, 0)$ and $(2, 0)$ and the line segment from $(-1, 0)$ to $(1, 0)$. State the Seifert-van Kampen Theorem, and use it to find the fundamental group of X.

B3. Prove the Brouwer fixed point theorem in dimension two: Every continuous map $f : D^2 \longrightarrow D^2$ has a fixed point.

C. **Mixed** (1 problem)

C1. Show that the following three conditions on a topological space X are equivalent:

i. Every continuous map $S^1 \longrightarrow X$ is null homotopic.

ii. Every continuous map $S^1 \longrightarrow X$ extends to a continuous map $D^2 \longrightarrow X$.

iii. The fundamental group $\pi_1(X, x_0)$ is trivial for all $x_0 \in X$.

C2. Let $X = S^1 \vee S^1$ be a wedge of two circles (i.e. two circles joined at a single point; a figure 8). Exhibit both a regular and an irregular 4-fold covering of X. Does X have an irregular 2-fold covering? (Explain.)
