Core Exam II
Graph Theory
Fall, 2002

Instructions: Solve any five from among the following seven problems. Submit only the five selected problems. You have 3 and 1/2 hours to complete this test. Good luck!

Note: Graphs are finite, undirected, and have no loops and no multiple edges.

1. Let G be a connected graph on n vertices, and let k be a positive integer less than n. Is it always true that there is a k-element subset X of $V(G)$ such that $G - X$ is connected?

2. Let G be a connected graph not having P_4 or C_3 as an induced subgraph. Prove that G is a complete bipartite graph.

3. Let G be a regular graph with a cut-vertex. Prove that $\chi'(G) > \Delta(G)$.

4. Use König-Egerváry Theorem to prove that every bipartite graph G has a matching of size at least $|E(G)|/\Delta(G)$. Use this to conclude that every subgraph of $K_{n,n}$ with more than $(k-1)n$ edges has a matching of size at least k.

5. For $n \geq 3$, let G be an n-vertex graph such that every graph obtained by deleting one vertex is a tree. Determine $|E(G)|$ and use it to determine G itself.

6. Suppose G is a 4-regular, planar, 2-connected graph. Is G necessarily Hamiltonian?

7. Without using the Four Color Theorem, prove that every planar graph with at most 12 vertices is 4-colorable.