Graph Theory Core II Comprehensive Exam

Solve any six from among the following eight problems. Submit only the six selected problems. You have 3 and 1/2 hours to complete this test. Good luck!

Note: Graphs are finite, undirected, and have no loops and no multiple edges.

- 1. Prove or disprove: Every 5-regular 3-connected planar graph is a triangulation.
- 2. Prove or disprove that the following two statements (a) and (b) are equivalent for all graphs with at least two edges.
 - (a) Every two vertices lie on a common cycle.
 - (b) Every two edges lie on a common cycle.
- 3. Use Euler's Formula to prove that the chromatic number of a graph embeddable in the projective plane is at most 6.
- 4. Suppose G is a connected 3-regular graph with a cut-edge. What are the possible values of the chromatic index of G?
- 5. Prove that every planar graph is a union of three forests.
- 6. A tree is a *caterpillar* if it has a path P such that every vertex of the tree either lies on P, or is a neighbor of a vertex of P. Find all minor-minimal trees that are not caterpillars.
- 7. Use Brooks' Theorem to give a short proof of a special case of Vizing's Theorem: If $\Delta(G) = 3$, then $\chi'(G) \leq 4$.
- 8. Find a graph without a cut-edge that has both a 3-flow and a cut of exactly 3 edges.