Core Exam II
Graph Theory
Fall, 2004

Instructions: Solve any five from among the following seven problems. Submit only the five selected problems. You have 3 and 1/2 hours to complete this test. Good luck!

Note: Graphs are finite, undirected, and have no loops and no multiple edges.

1. Let G be a bipartite graph with bipartition (X, Y), and let A be the set of vertices of maximum degree in G. Show that G has a matching that meets all vertices of $A \cap X$.

2. Suppose G is a 3-regular Hamiltonian graph on at least six vertices. What are the possible values for the chromatic number and for the chromatic index of G?

3. Prove that if G is k-connected, where $k \geq 2$, then every set of k vertices is contained in a cycle.

4. Prove that the set of edges in a connected graph G forms a spanning tree of G if and only if the duals of the remaining edges form a spanning tree of G^*.

5. Suppose G is a plane graph with minimum degree three and girth five. What is the smallest possible number of vertices of degree three in G?

6. Prove that a cubic graph has a 3-flow if and only if it is bipartite. Give an example of a 3-flow in the cube.

7. State the vertex and the edge versions of Menger's Theorem. Prove the edge version using the vertex version.