Name:

Graph Theory Core II Comprehensive Exam

Solve any six from among the following eight problems. Explain all of your answers. Submit only the six selected problems. You have 3 and 1/2 hours to complete this test. Good luck!

Note: Graphs are finite, undirected, and have no loops and no multiple edges.

- 1. Suppose G is a graph of order $n \ge 4$ and minimum degree at least n-2. Is G necessarily Hamiltonian? What if the two "minimum" is replaced by "average"?
- 2. Suppose G is a connected 4-regular graph of order at least six. What is the maximum possible value of the chromatic number of G? Give an example of such a graph G with the maximum possible chromatic number.
- 3. Use Euler's Formula to determine the maximum number of edges in a bipartite planar graph of order n.
- 4. Find the flow number of K_5 . Give an example of a non-zero k-flow on K_5 for the smallest possible value of k.
- 5. Let $k \ge 2$. Show that every k-connected graph of order at least 2k contains a cycle of length at least 2k.
- 6. Suppose G_1 and G_2 are connected plane graphs such that the numbers of vertices, edges, and faces of G_1 are the same as the respective numbers of vertices, edges, and faces of G_2 . Are G_1 and G_2 necessarily isomorphic?
- 7. Suppose k is a positive integer. Is there a k-regular graph with a cut-vertex and chromatic index k?
- 8. Does K_5 admit a cellular embedding into the torus? If yes, give a rotation system; if not, explain why.