Graph Theory Core II Comprehensive Exam

Solve any five from among the following seven problems. Submit only the five
selected problems. You have 3 and 1/2 hours to complete this test. Good luck!

Note: Graphs are finite, undirected, and have no loops and no multiple edges.

1. Find, with proof, all 3-regular graphs on fewer than eight vertices.

2. Suppose G is a connected 3-regular graph with a cut-edge. What are the possible values
   of the chromatic number of G?

3. Suppose G is a connected 3-regular graph with a cut-edge. What are the possible values
   of the chromatic index of G?

4. Without invoking the Four Color Theorem, prove that a plane graph is 4-colorable if its
   dual is Hamiltonian.

5. Suppose G is an r-regular bipartite graph and F is a set of some $r-1$ edges of G. Prove
   that $G \backslash F$ has a perfect matching.

6. What is the smallest number of degree-three vertices that a planar graph of circumference
   four can have?

7. Assume that the Tutte Four-Flow Conjecture holds to derive the Four Color Theorem.