January 12, 2007 Name: ______________________

Graph Theory Core II Comprehensive Exam

Solve any six from among the following eight problems. Submit only the six selected problems. You have 3 and 1/2 hours to complete this test. Good luck!

Note: Graphs are finite, undirected, and have no loops and no multiple edges.

1. Prove or disprove:
   Every 5-regular 3-connected plane graph is a triangulation.
2. Suppose G is a 2-connected graph that is not K_3, and e is an edge of G. Show that at least one of $G\backslash e$ and G/e is also 2-connected.
3. Use Euler's Formula to prove that the chromatic number of a graph embeddable in the projective plane is at most 6.
4. Prove that a graph with every vertex degree even has no cut-edge.
5. Suppose G is a k-regular bipartite graph for some positive integer k. Prove that G has a perfect matching. Is the "bipartite" assumption necessary when k is odd?
6. Show that every cubic Hamiltonian graph has at least three Hamilton cycles.
7. Suppose G is a triangle-free planar graph. Without invoking the Four Color Theorem or the Grötzsch's Theorem, show that G is 4-colorable.
8. Use only elementary arguments (without invoking any theorem on flows) to determine the flow number of K_4 .