Instructions: *Do any* **five** *of the following problems. Justify your statements and calculations. Be sure to write the number for each problem you work out and write your name clearly at the top of each page you turn in for grading. You have three and a half hours.* Good luck!

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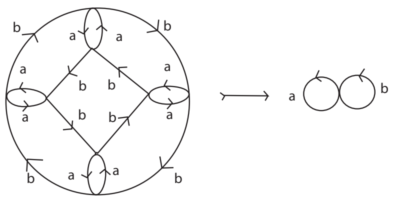
1. Let X be the union of the 3-coordinate axes in $\mathbb{R}^3$ and the unit sphere in $\mathbb{R}^3$. Calculate the fundamental group of X.

2. Let X be a path connected and locally path connected space whose fundamental group is finite. Show any continuous map from X to a Klein bottle is homotopic to a constant map.

3. What is the fundamental group of a Mobius band? Determine if the boundary circle of a Mobius band is or is not a retract of the Mobius band.

4. For each positive integer n, let C_n be a circle of radius $1/n$ with center $(0,(1/n)) \in \mathbb{R}^2$. The union of these circles is called the Hawaiian earring. Does it have a simply connected covering space?

5. Consider the covering space of the wedge of two circles indicated in the following figure. Give a presentation for the group of covering transformations with 2 generators and 3 relations. How many elements does the group of covering transformations have?



6. Show that a retract of a contractible space is contractible.

7. Let n be a positive integer different from 2. Show $\mathbb{R}^2$ is not homeomorphic to $\mathbb{R}^n$.